Polarized radiation transfer in multidimensional models of the solar atmosphere

Tanausú del Pino Alemán

19^{th} September 2019

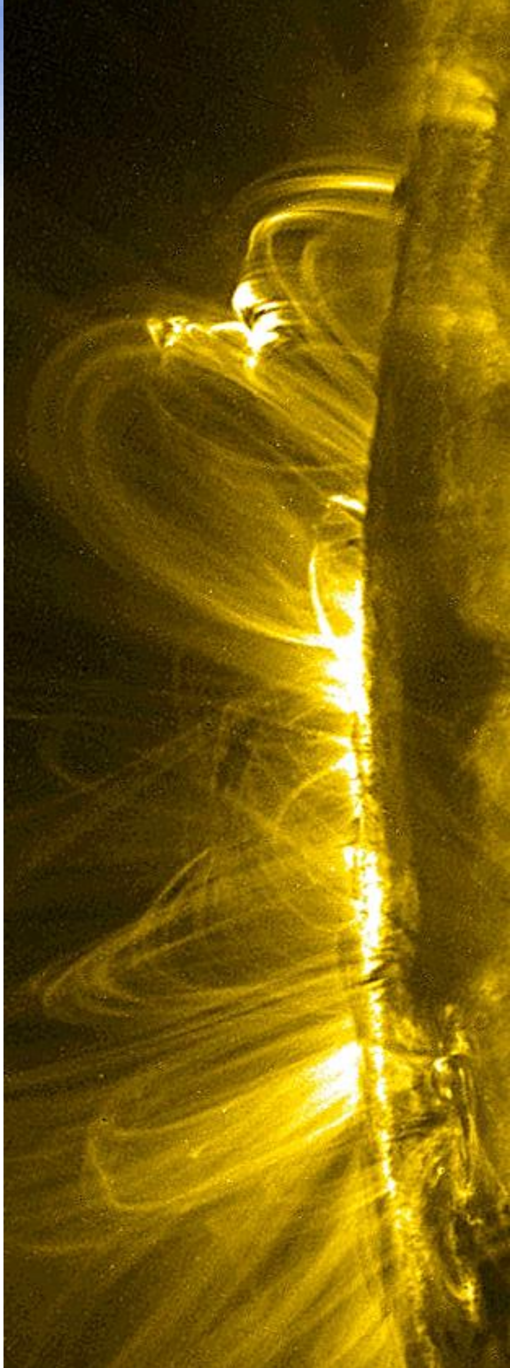
Introduction

- Magnetic fields play a key role in the physics of the solar atmosphere

- Responsible of the solar activity

- Forms the plasma structures of the outer solar atmosphere

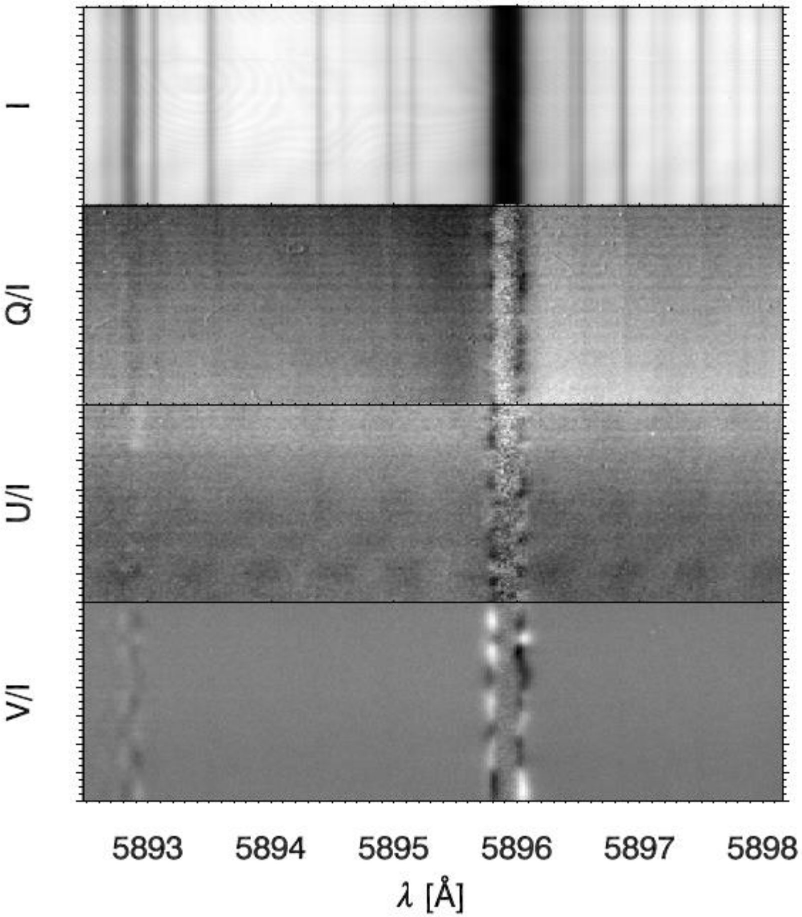
- Key to explain the existence of a hot (1 million degrees) corona



From the **TRACE** satellite

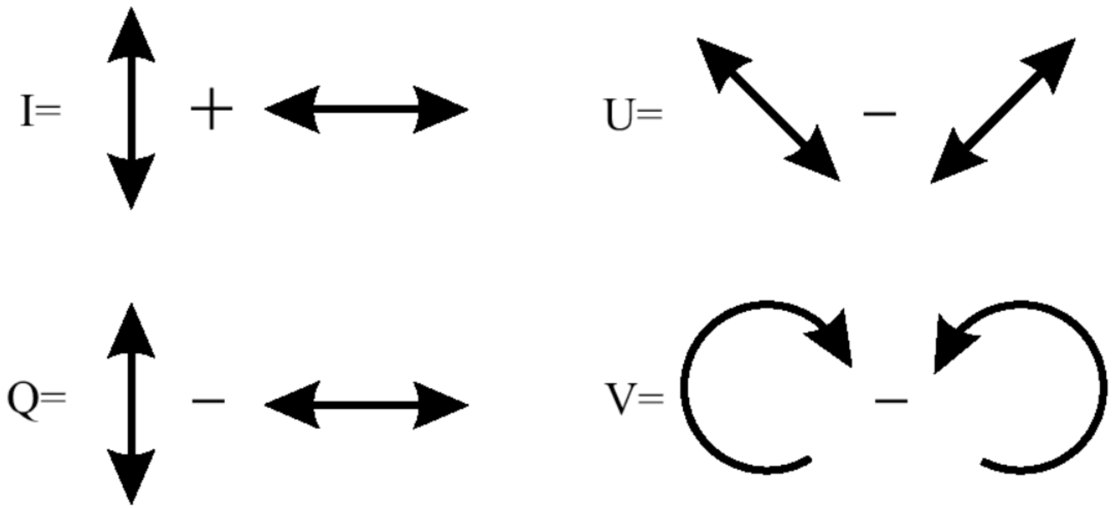
Introduction

Na I D1 slit observation with ZIMPOL@IRSOL



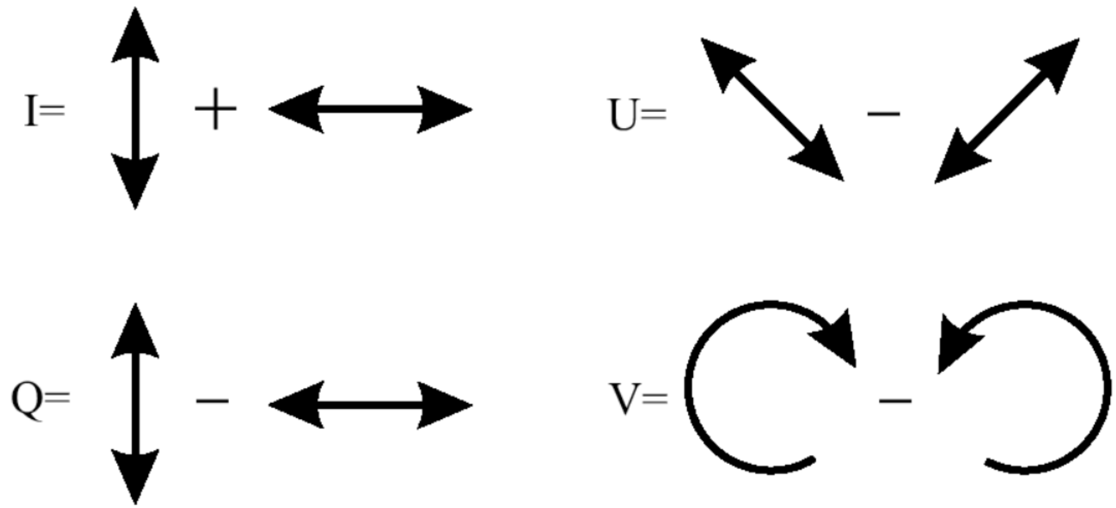
- Cannot be directly measured
- We measure electromagnetic radiation (photons)
- The measured radiation has information about the properties of the emitting plasma

Introduction



- Polarization is present when there is no symmetry:
    - Scattering: radiation pumping by anisotropic radiation
    - Zeeman effect: energy splitting of degenerate atomic levels
    - Hanle effect: relaxation of quantum coherences

Introduction



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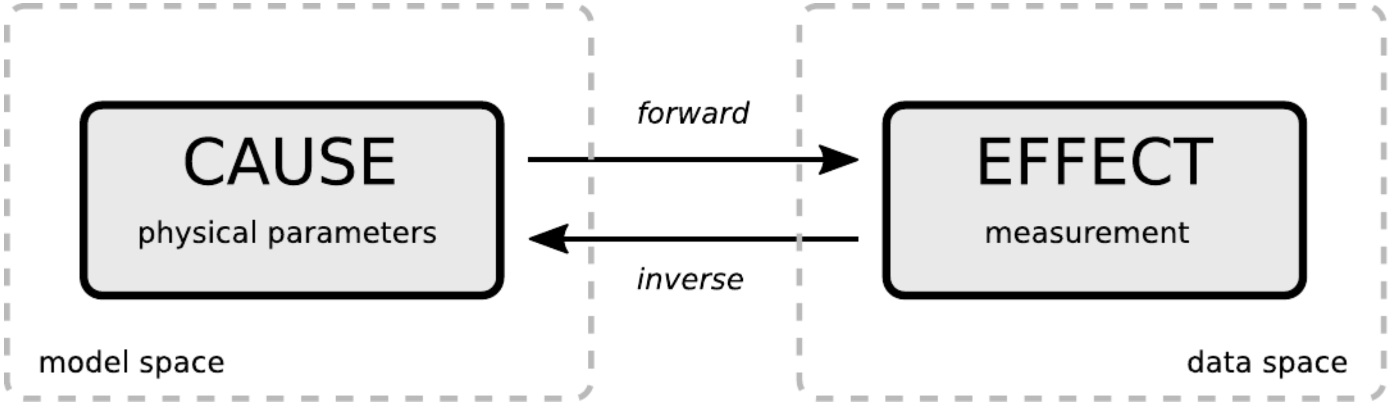
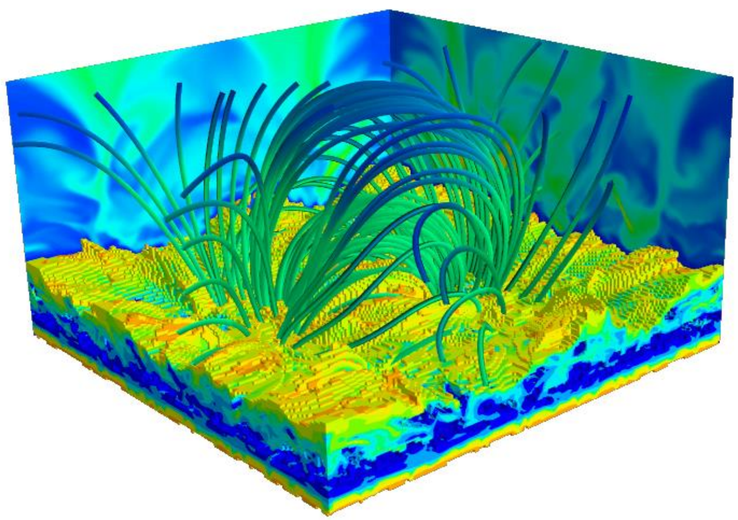
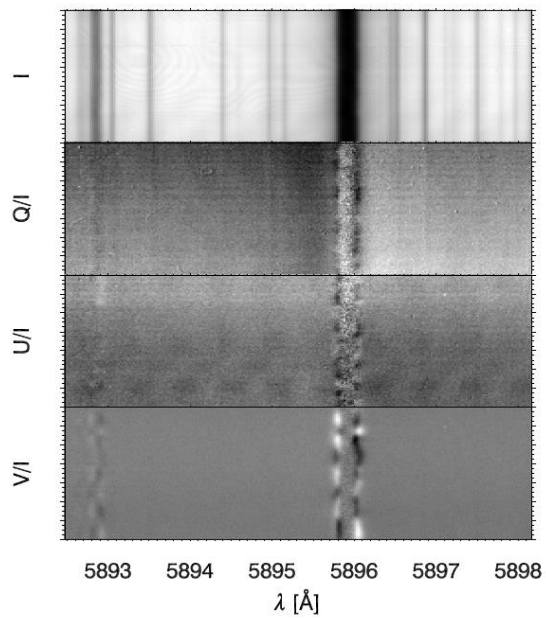
Magnetic effects

Introduction





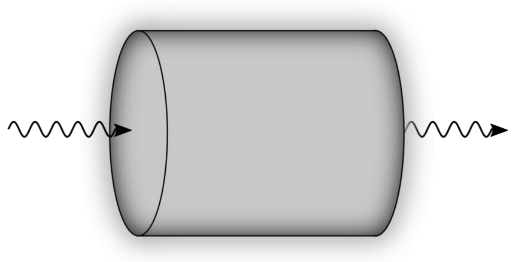


Na I D1 slit observation with ZIMPOL@IRSOL

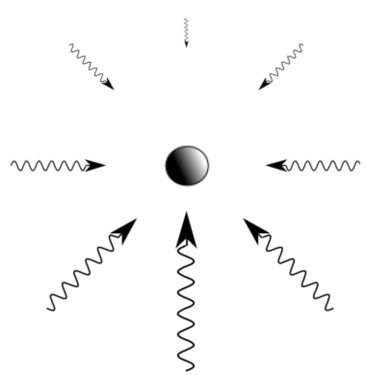
The Forward Problem
Radiation Transfer

Radiation Transfer

- Describes the radiation-matter interaction
- Two parts:



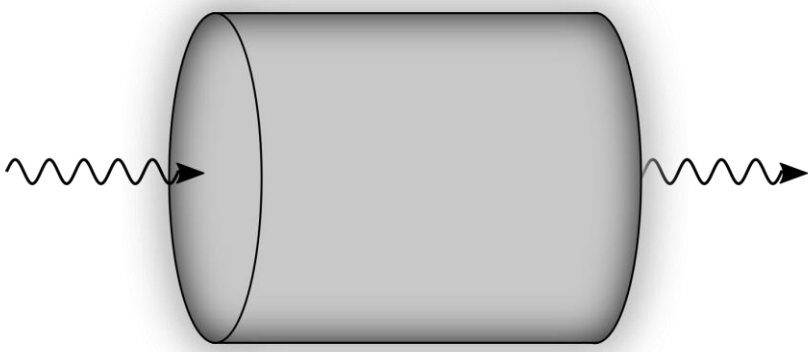
- How is the radiation propagated through a medium



- How are the atoms excited within the atmospheric radiation field

Radiation Transfer Equation

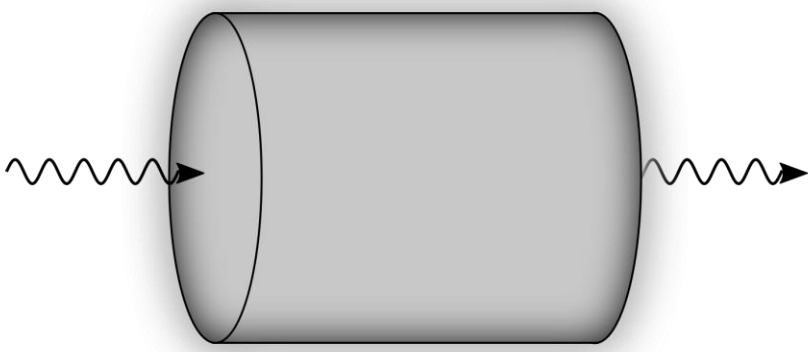
- How the radiation is modified along its propagation



$$\frac{d}{ds}\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = -\begin{pmatrix} \eta_I & \eta_Q & \eta_U & \eta_V \\ \eta_Q & \eta_I & \rho_V & -\rho_U \\ \eta_U & -\rho_V & \eta_I & \rho_Q \\ \eta_V & \rho_U & -\rho_Q & \eta_I \end{pmatrix}\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} + \begin{pmatrix} \epsilon_I \\ \epsilon_Q \\ \epsilon_U \\ \epsilon_V \end{pmatrix}$$

Radiation Transfer Equation

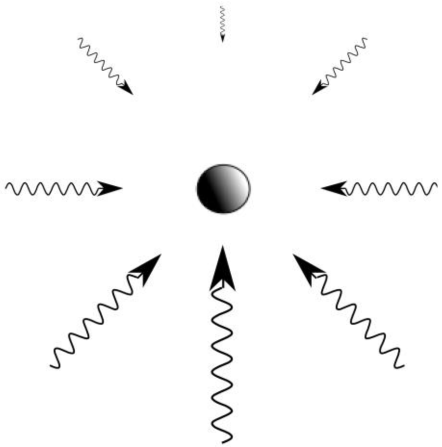
- How the radiation is modified along its propagation



$$\frac{d\mathbf{S}}{ds} = -\hat{\mathbf{K}} \cdot \mathbf{S} + \epsilon$$

Statistical Equilibrium Equations

- How the atom is excited within a radiation field



$$
\begin{aligned}
\left[\frac{d}{dt}\,{}^{L}\rho^{K}_{Q}(J,J')\right]_{\mathrm{Rad}} &= \sum_{L_\ell J_\ell J'_\ell K_\ell Q_\ell} {}^{L_\ell}\rho^{K_\ell}_{Q_\ell}(J_\ell,J'_\ell)\,\mathbb{T}_{\mathrm{A}}(LJJ'KQ,L_\ell J_\ell J'_\ell K_\ell Q_\ell) \\
&+ \sum_{L_u J_u J'_u K_u Q_u} {}^{L_u}\rho^{K_u}_{Q_u}(J_u,J'_u)\big[\mathbb{T}_{\mathrm{E}}(LJJ'KQ,L_u J_u J'_u K_u Q_u)+\mathbb{T}_{\mathrm{S}}(LJJ'KQ,L_u J_u J'_u K_u Q_u)\big] \\
&- \sum_{J'' J''' K' Q'} {}^{L}\rho^{K'}_{Q'}(J'',J''')\big[\mathbb{R}_{\mathrm{E}}(LJJ'KQJ''J'''K'Q')+\mathbb{R}_{\mathrm{S}}(LJJ'KQJ''J'''K'Q')+ \\
&\qquad \mathbb{R}_{\mathrm{A}}(LJJ'KQJ''J'''K'Q')\big]
\end{aligned}
$$

Statistical Equilibrium Equations

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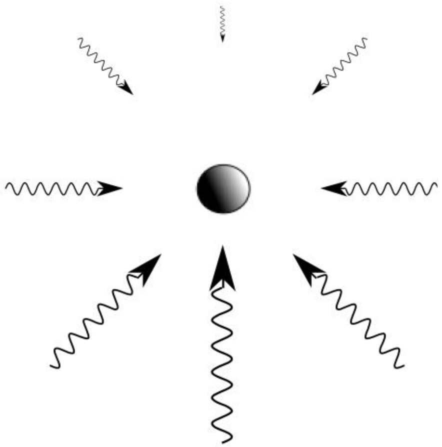

$$\frac{d\rho_i}{dt} = \sum_{j \neq i} R_{ji}\rho_j - \sum_j R_{ij}\rho_i = 0$$

Radiation Transfer

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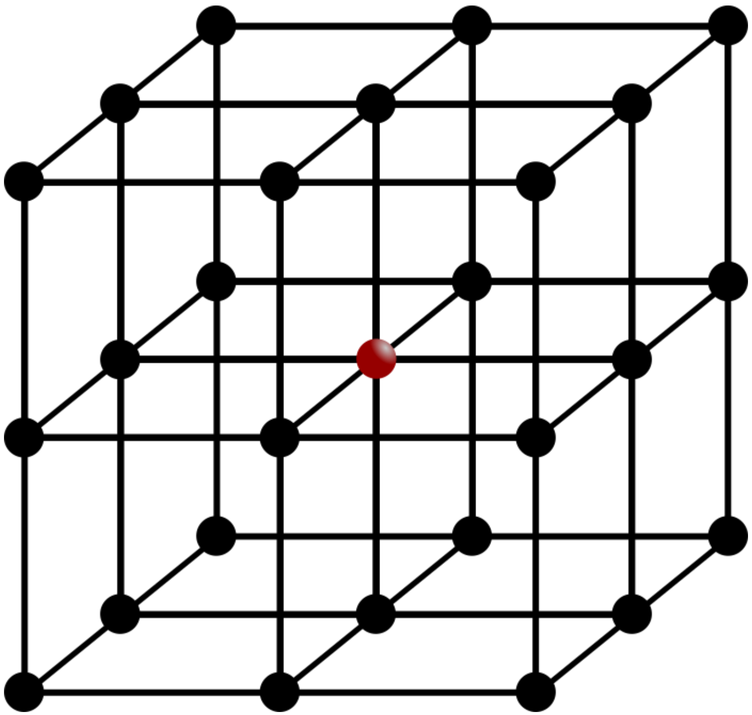
- Coupled
- Non-linear
- Non-local
- Most difficult (costly) problem to solve in solar physics

Radiation Transfer

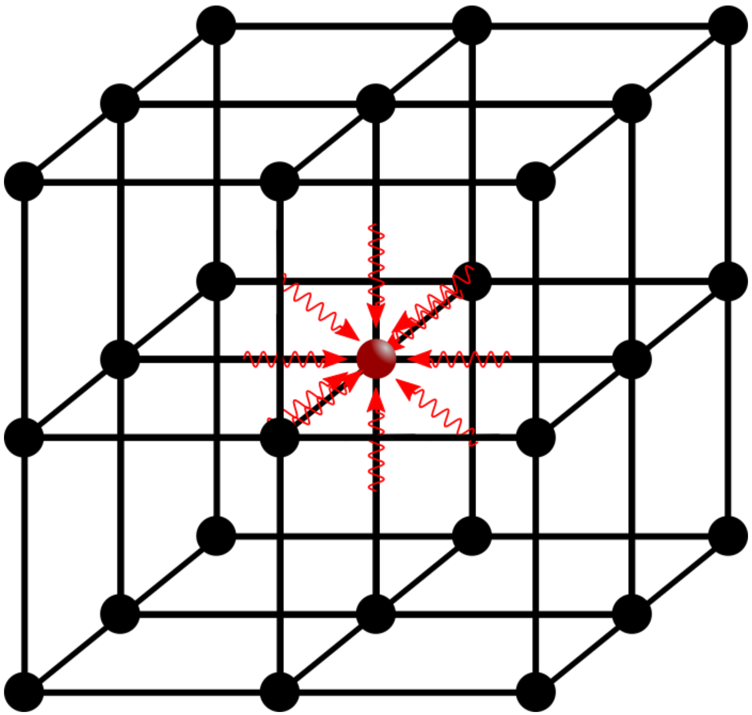


- Spatial nodes:

  $nx, ny, nz \sim 10^2$

  $nx \cdot ny \cdot nz \sim 10^6$

Radiation Transfer



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- Directions:
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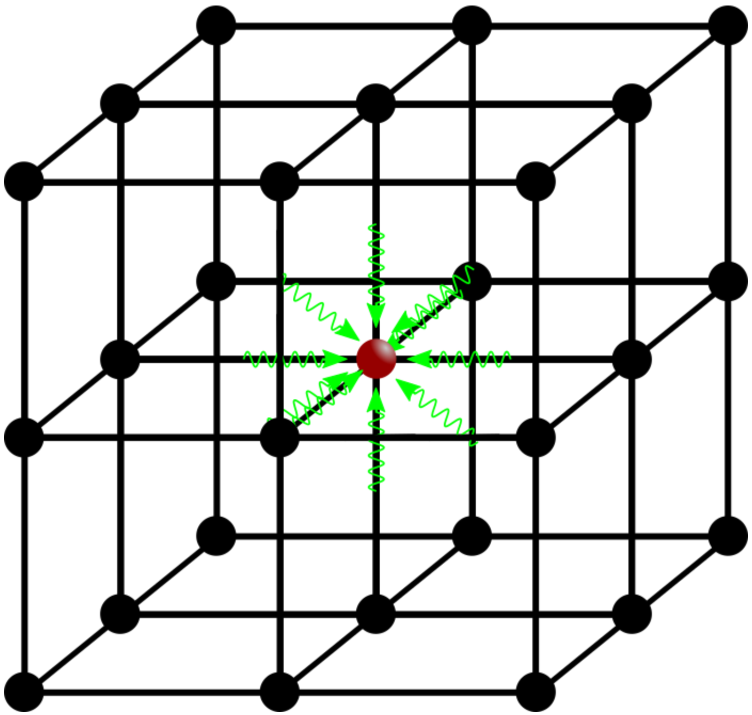
Radiation Transfer



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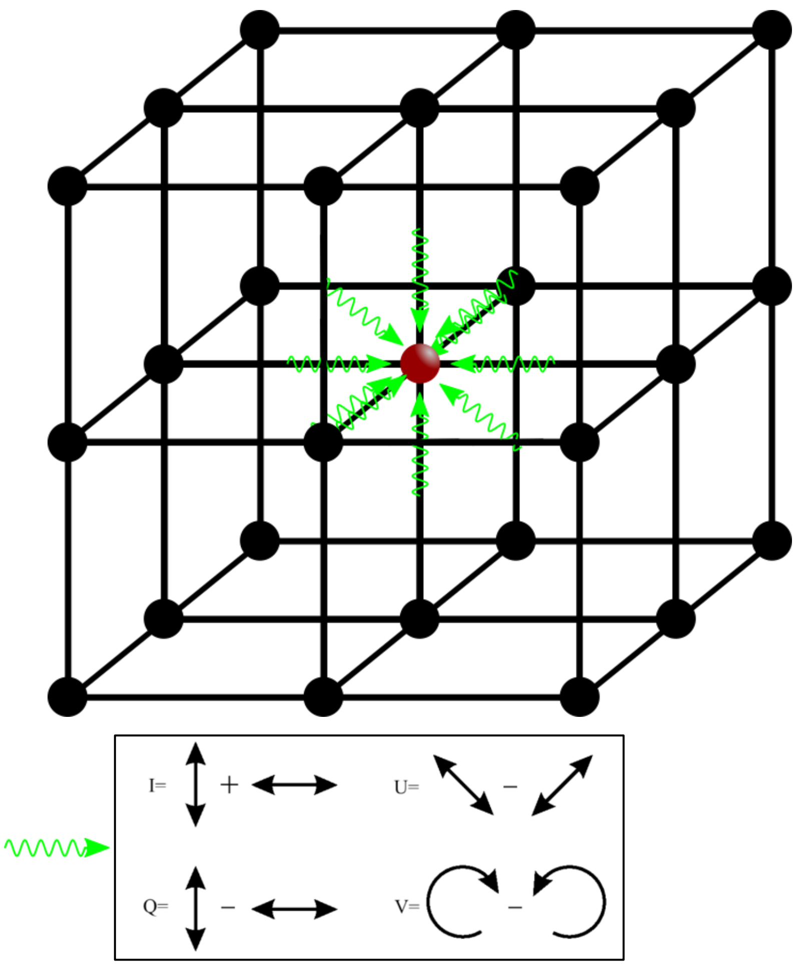

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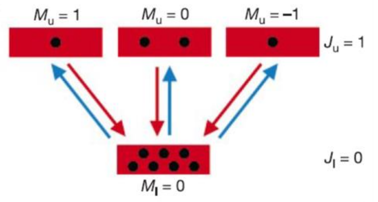
- Polarization:
  $ns \sim 4$

Radiation Transfer

- Simplest problem: two-level atom $J_l = 0; J_u = 1$



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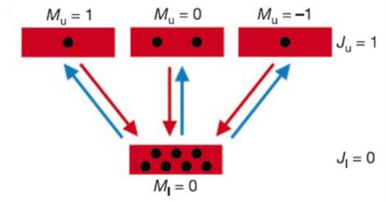

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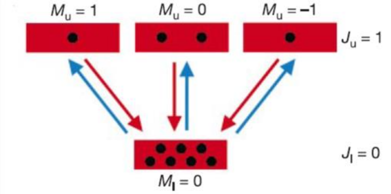
- Statistical equilibrium:
    - 10 unknowns per spatial node → 10^7

Radiation Transfer



- Simplest problem: two-level atom $J_l = 0; J_u = 1$
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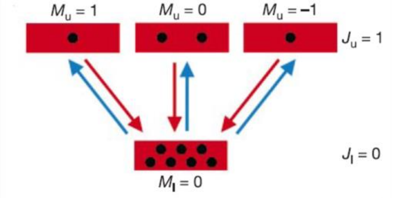
- Radiation field:
    - Polarization in every node, frequency, and direction, 10^{10} unknowns

Radiation Transfer



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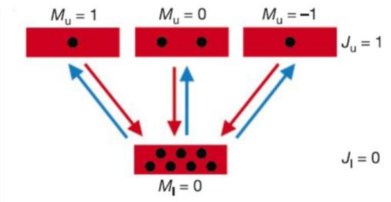

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Parallelization is a **must**

PORTA

PORTA

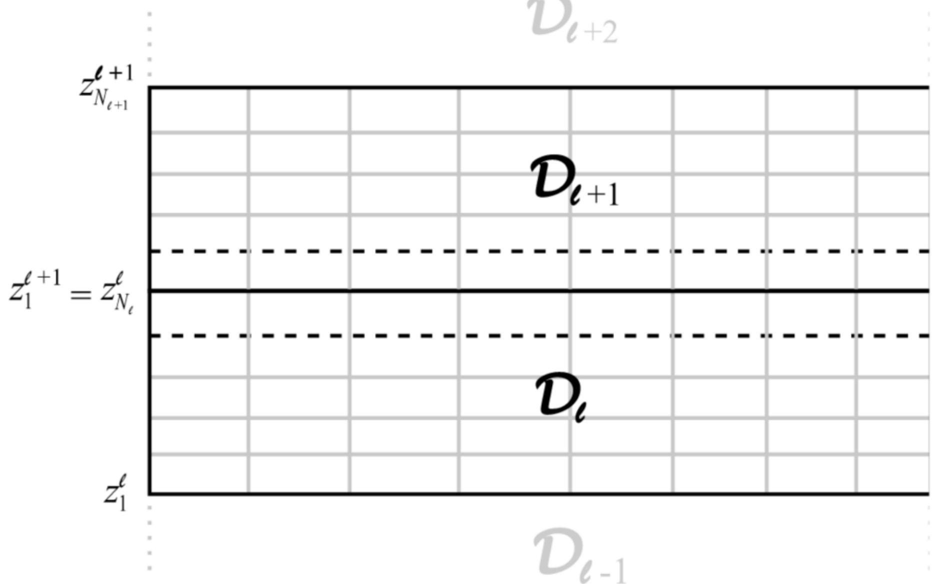
- Library to solve the problem of the generation and transfer of polarized radiation in 3D atmospheres
  Štěpán & Trujillo Bueno (2013)

- Modules to solve specific problems

- Almost linear scaling with #CPU

PORTA

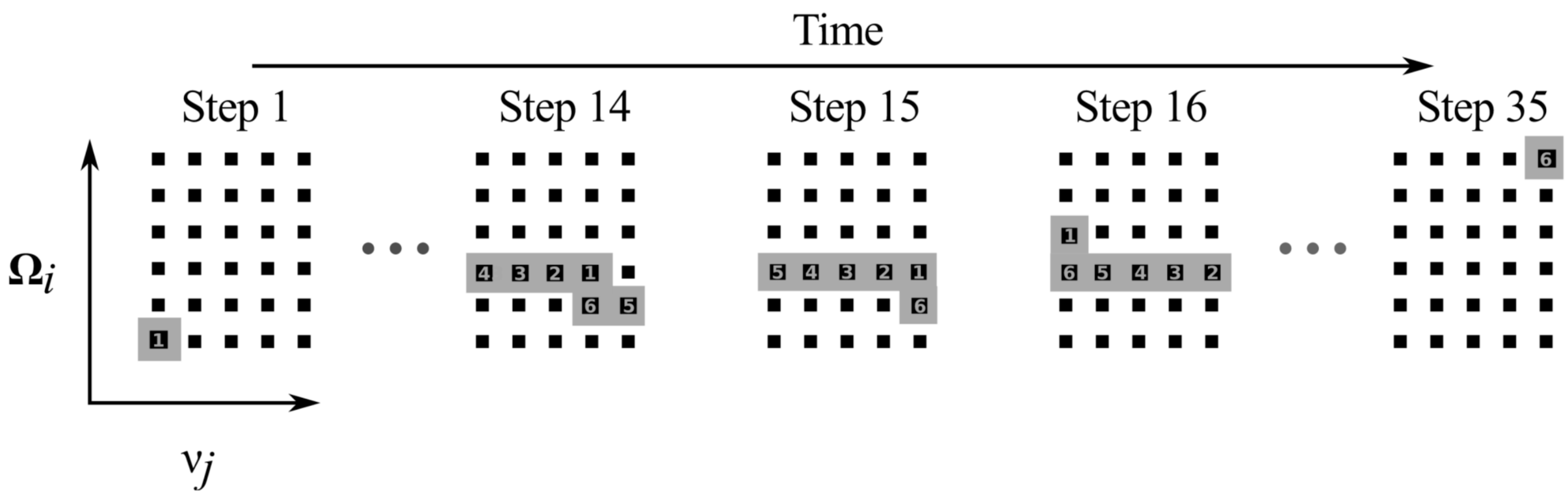
- Domain decomposition:
  - Distributes work
  - Eases memory constrains



Štěpán & Trujillo Bueno (2013)

PORTA

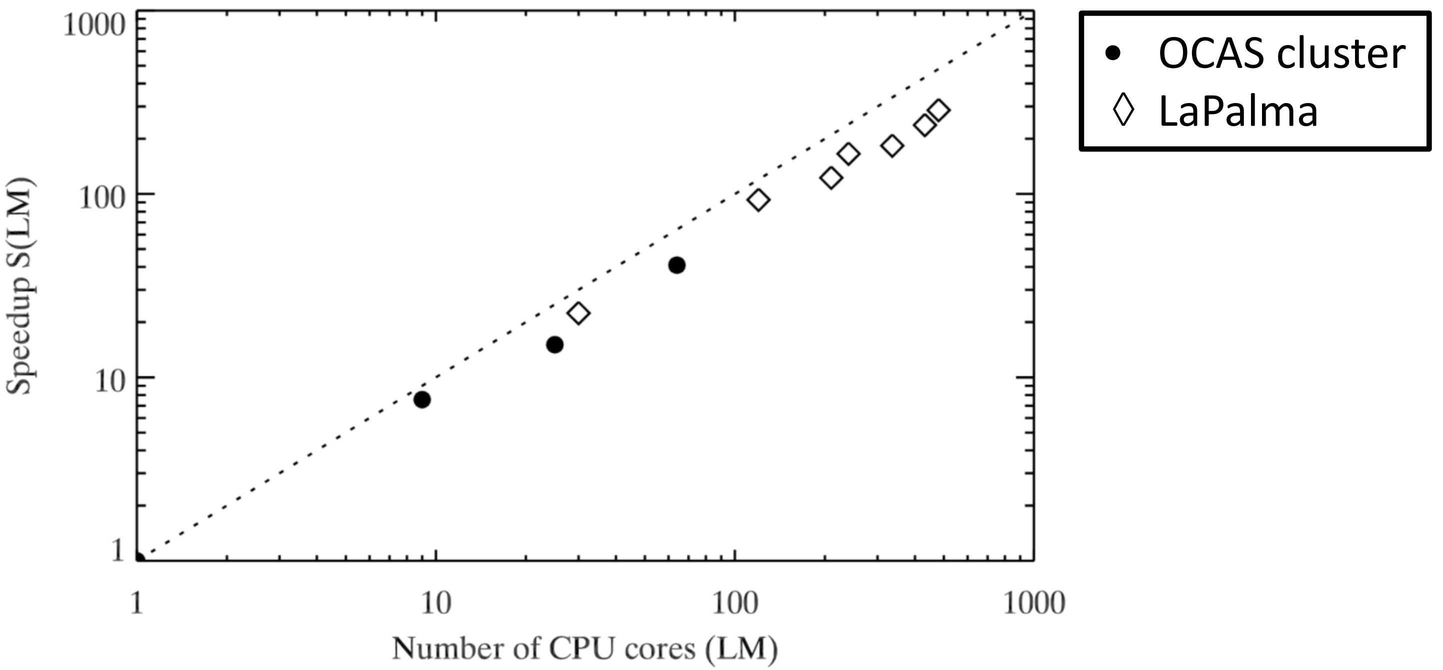
- Snake algorithm:



Štěpán & Trujillo Bueno (2013)

PORTA

- Linear scaling:



Štěpán & Trujillo Bueno (2013)

PORTA

- Close to be public with the modules:
    - Coherent scattering (Jaume Bestard, J., del Pino Alemán, T., and Štěpán, J.)
    - General two-level (Štěpán, J.)
    - General multi-level (del Pino Alemán, T.)

Applications
in MareNostrum

PORTA

- Every application uses the PORTA code
- But every application is different
    - Preliminar investigations (1D theoretical studies)
    - Preliminar computations (preparation of 3D models and computation of intermediate quantities)
    - Different modules
- I will only talk about the very final results of some investigations
    - And only about one or two results of the chosen ones

PORTA

- Hydrogen Lyman-α:
  - Theoretical study (Štěpán et al. (2015))
  - Diagnostics of CLASP data (Trujillo Bueno et al. (2018))
- Hydrogen Balmer-α:
  - Theoretical study (Jaume Bestard PhD. thesis, WIP)
- Calcium 4227 Å:
  - Theoreical study (Jaume Bestard PhD. thesis, WIP)
- Calcium H-K and infrared triplet:
  - Theoretical study (Štěpán and Trujillo Bueno (2016))
  - Comparison with observations (Jurčák et al. (2018))

PORTA

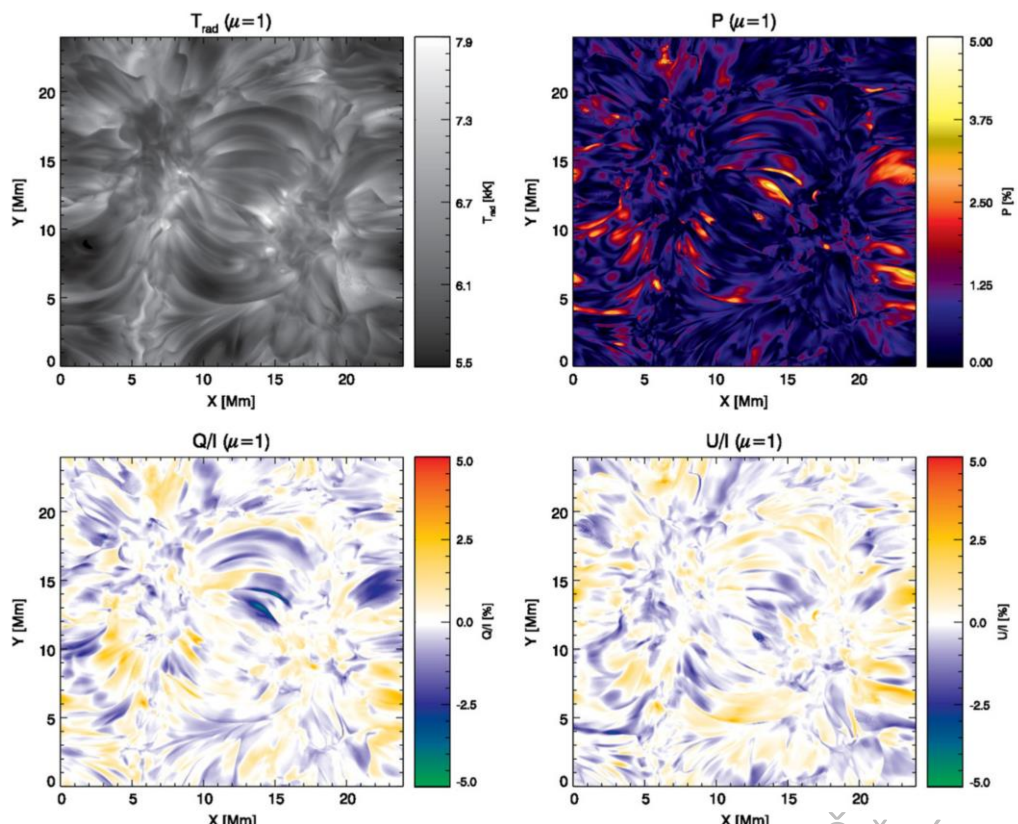
- Mg II k-line:
    - Theoretical study (del Pino Alemán PhD. thesis (2015))
- Sr I 4607 Å:
    - Theoretical study and comparison with observations (del Pino Alemán et al. (2018))
- Radiation transfer theoretical study:
    - Polarization with horizontal inhomogeneities (Tichý et al. (2015))

Some Results

Diagnostic of CLASP observations

Diagnostic of CLASP observations

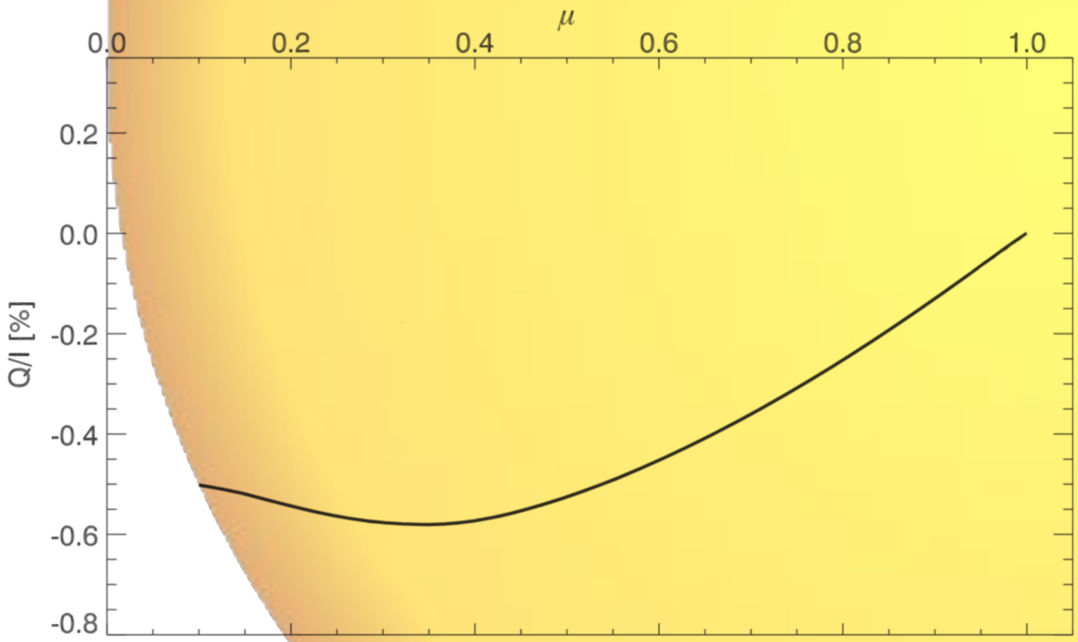
- We carried out the detailed radiation transfer modeling of the Lyman-α line



Štěpán et al. (2015)

Diagnostic of CLASP observations

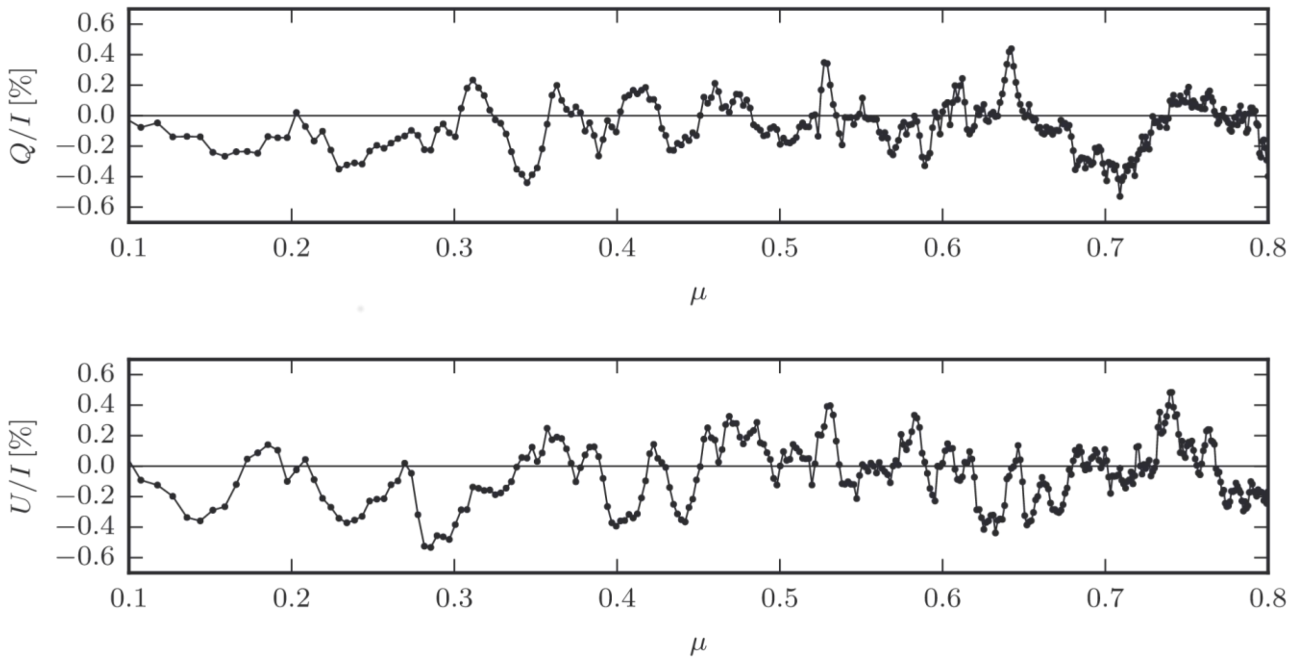
- Theory predicted center-to-limb variation of the linear polarization in the center of the Lyman-α line



Štěpán et al. (2015)

Diagnostic of CLASP observations

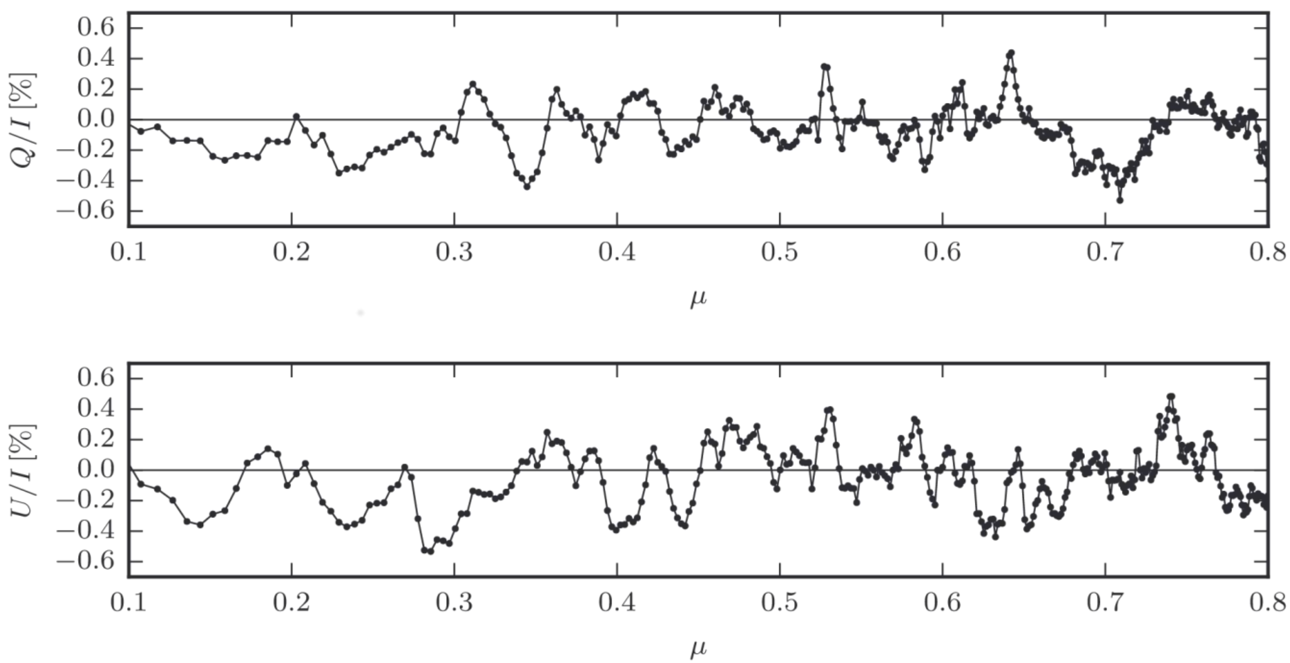
- But observations did not show any



Kano et al. (2017)

Diagnostic of CLASP observations

- But observations did not show any
- The 3D model could not reproduce the observations



Kano et al. (2017)

Diagnostic of CLASP observations

- What other properties should the 3D model have?

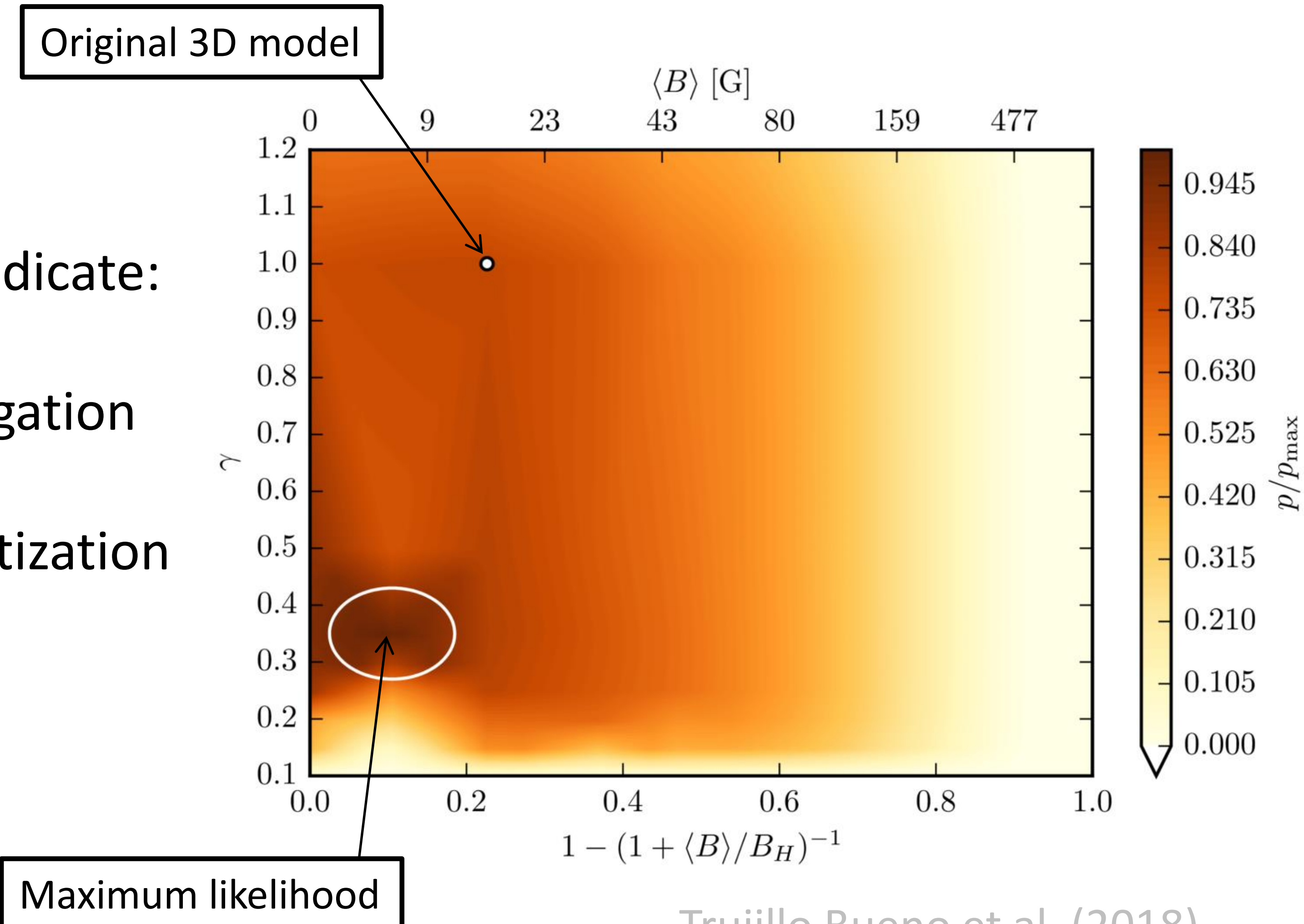
- We introduced two parameters:
    - Geometrical complexity (compression factor)
    - Magnetization (magnetic field strength factor)

- Bayesian approach: what is the combination of parameters with the most likelihood?

Diagnostic of CLASP observations

Observations indicate:

- More corrugation
- Less magnetization

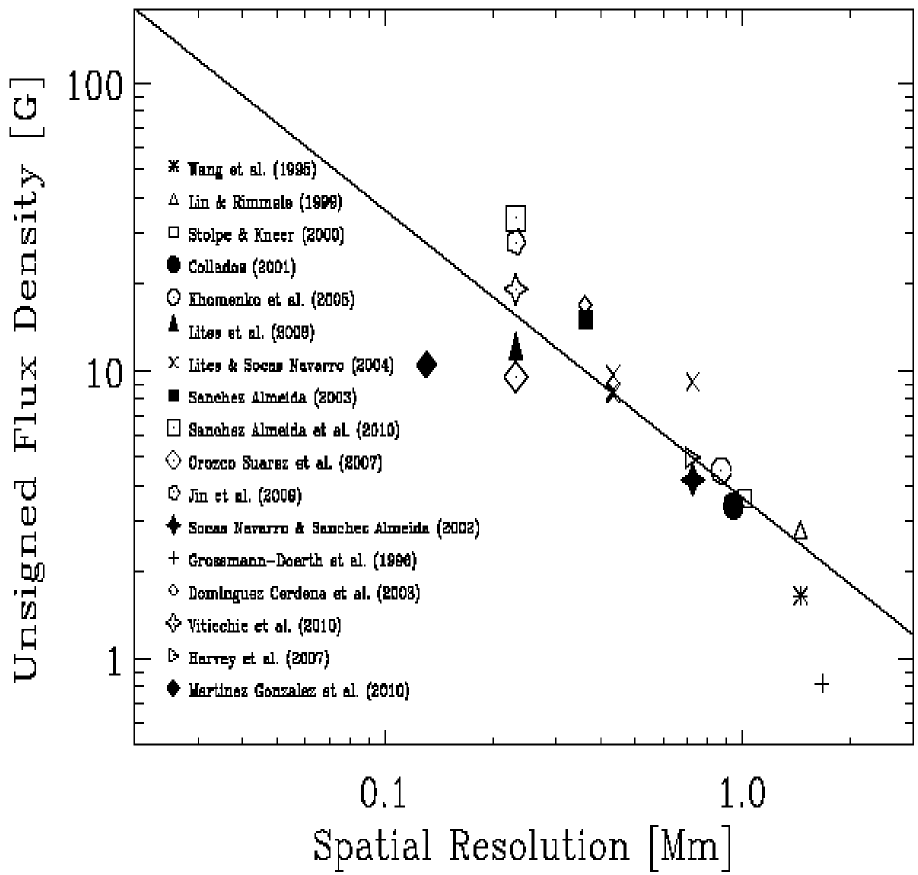


Trujillo Bueno et al. (2018)

Photospheric small scale magnetization

Photospheric small scale magnetism

- The photospheric magnetic field has structure at very small scales
- The resolution limit of solar observations limits our detection capability
- More field the better the resolution



Photospheric small scale magnetism

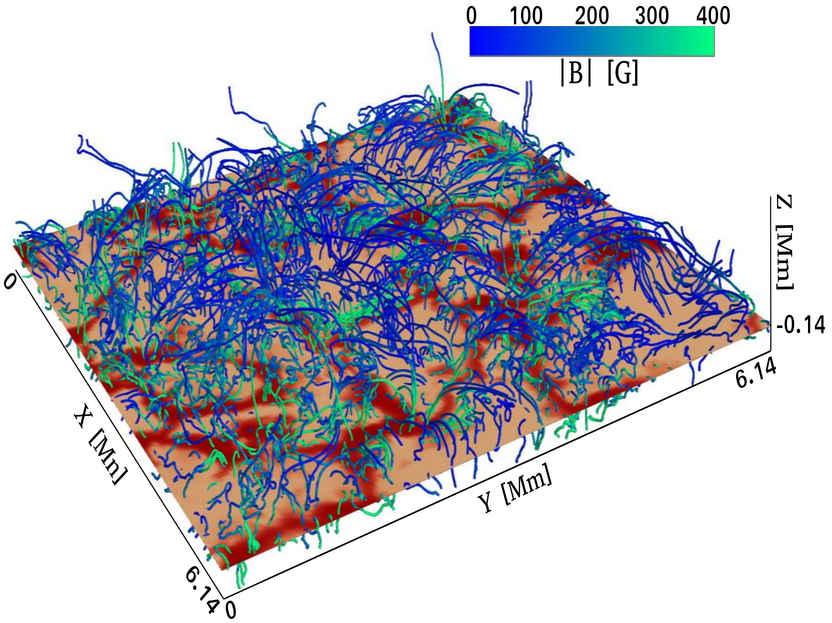
- Several authors with different approaches have provided estimations of <B>.

  - Stenflo (1982): >10G
  - Faurobert et al. (1995): 10-20G
  - Trujillo Bueno et al. (2004): 100G

Photospheric small scale magnetism

- All previous 3D (Hanle) calculations assumed non spatially resolved magnetic fields

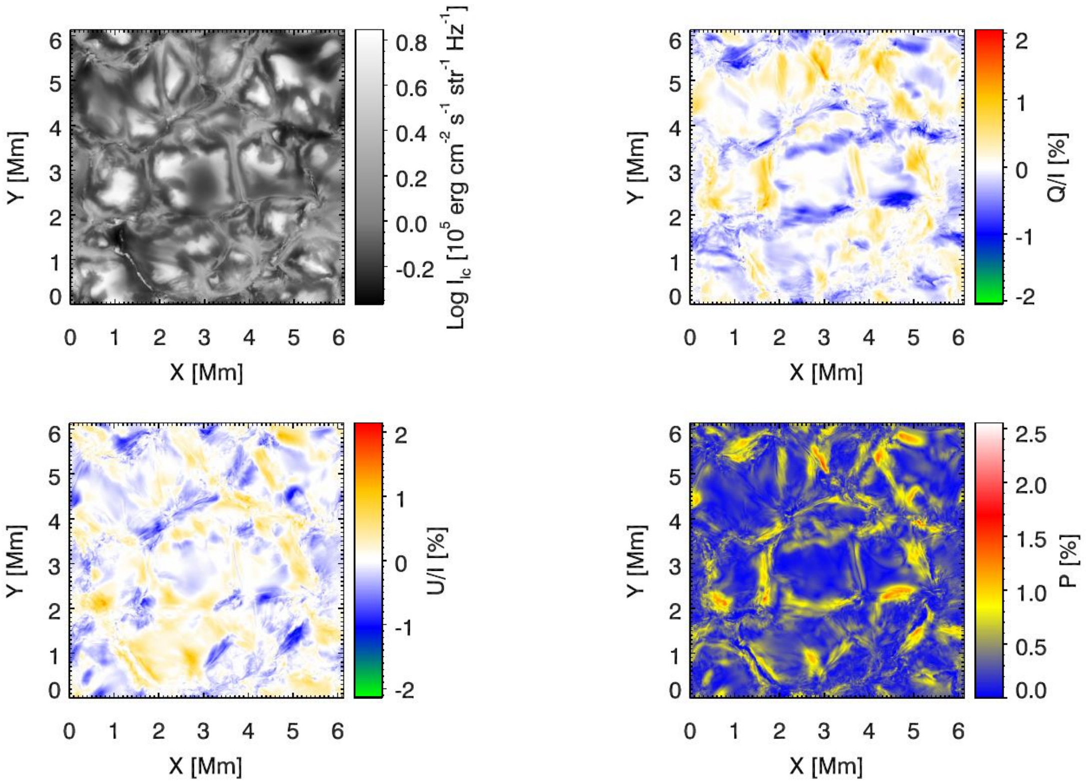
- Rempel (2014) provided 3D magneto-convection simulations with significant level of small-scale magnetic activity <B>≈170G at height ≈ 0km



del Pino Alemán et al. (2018)

Photospheric small scale magnetism

- We carried out the detailed radiation transfer modeling of Sr I 4607Å in this model



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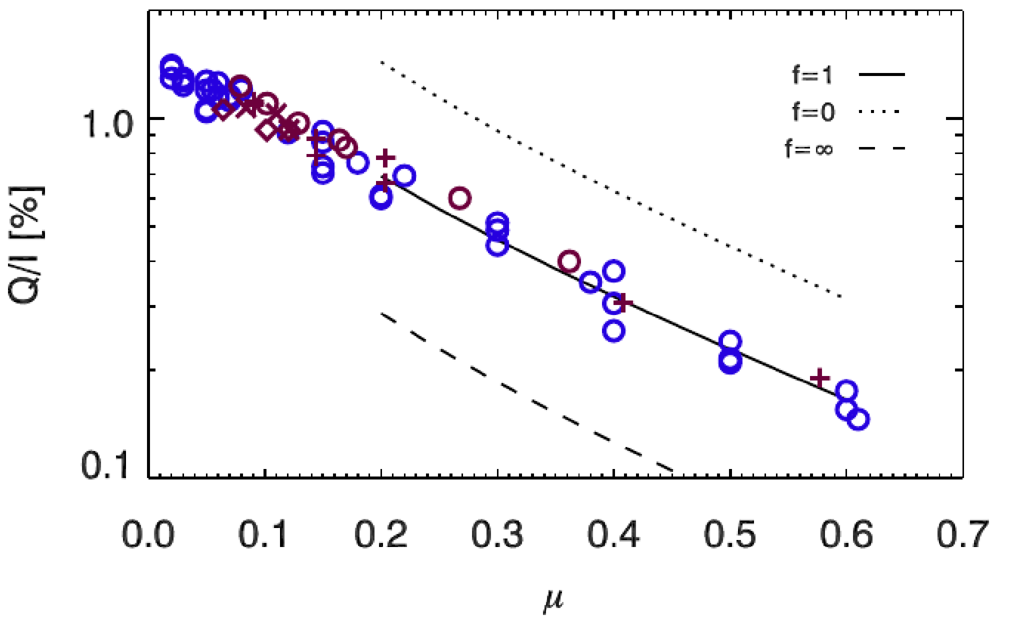
- Compare with center-to-limb variation observations



del Pino Alemán et al. (2018)

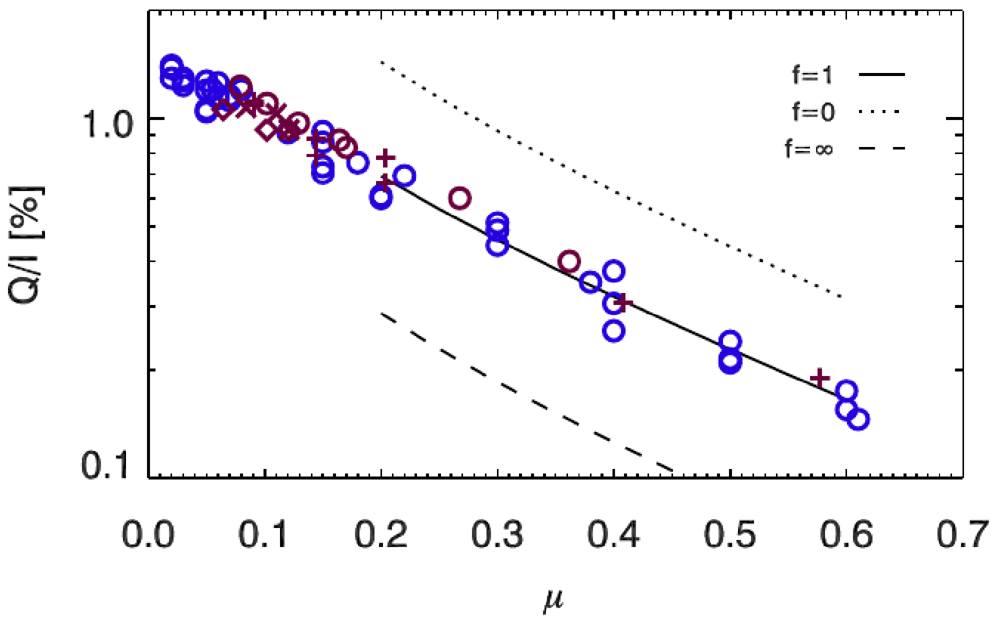
Photospheric small scale magnetism

- We carried out the detailed radiation transfer modeling of Sr I 4607Å in this model

- Compare with center-to-limb variation observations

- The level of magnetization is compatible with the observations: <B>≈170G at the surface

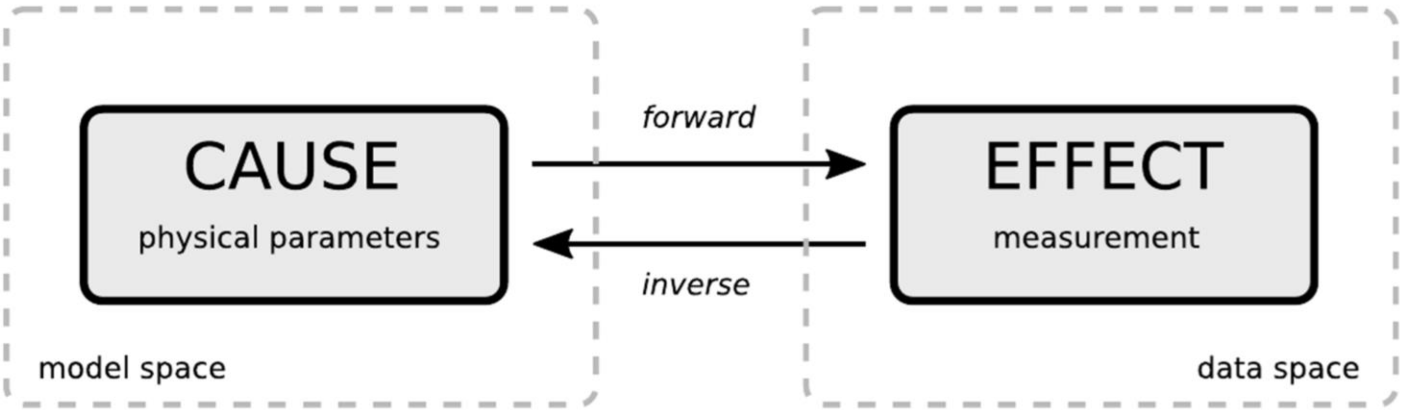
- (and a lot of other theoretical/statistical considerations I have no time to talk about now)

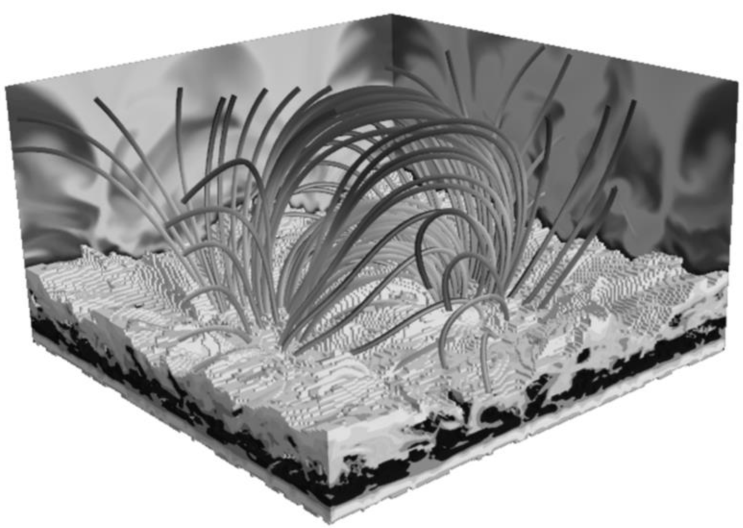


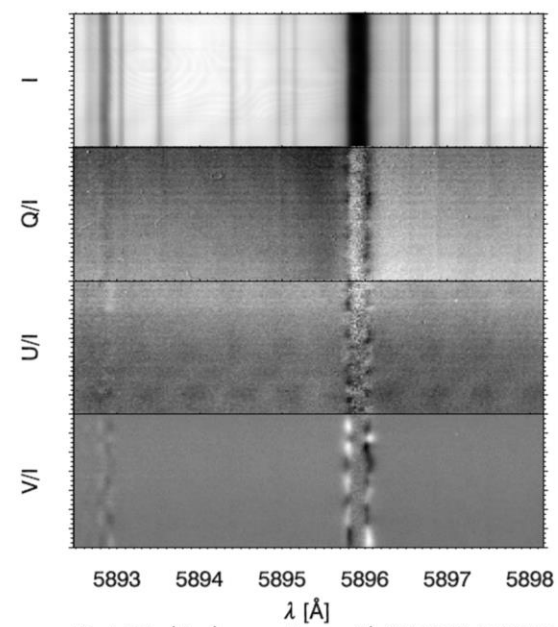
del Pino Alemán et al. (2018)

That is a lot of forward modeling but...

Introduction

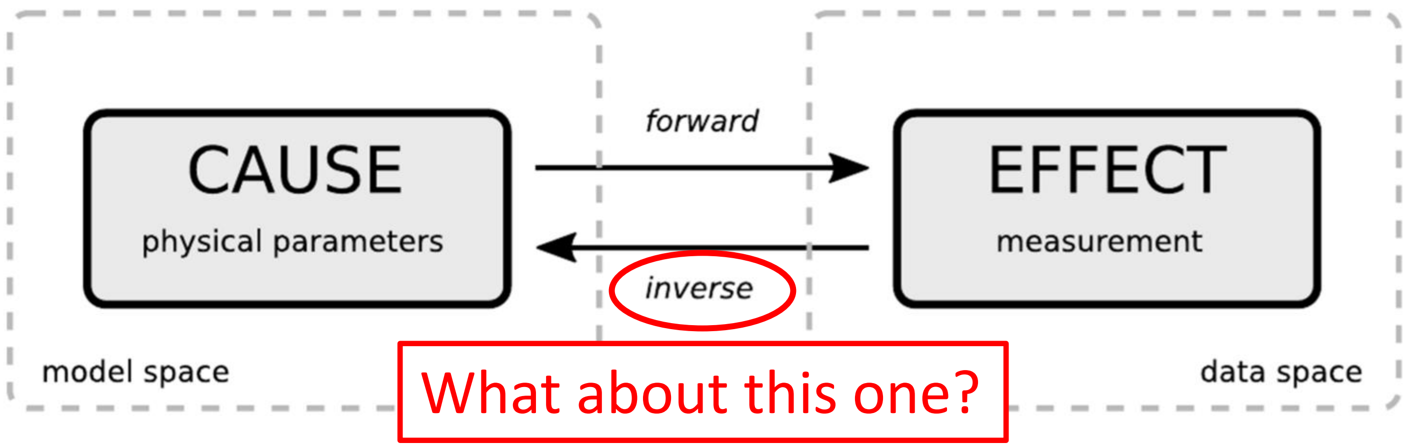


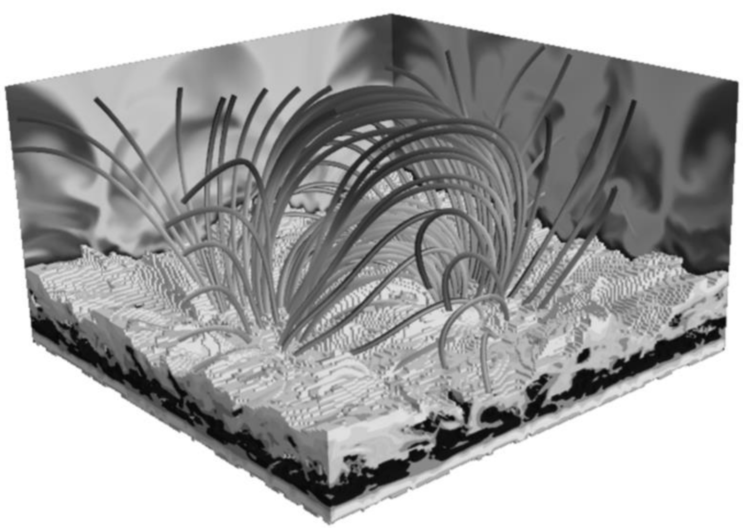


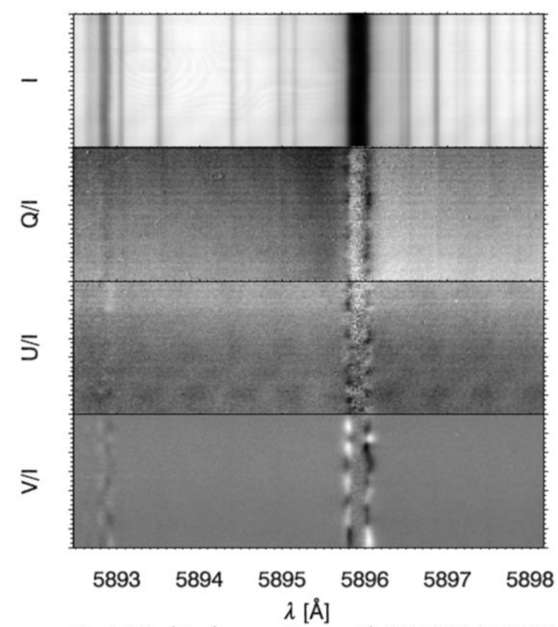


Na I D1 slit observation with ZIMPOL@IRSOL

Introduction







Na I D1 slit observation with ZIMPOL@IRSOL

The inversion problem

The inversion problem

- Find the x-parameters that fulfills

$$y = F(x) + \varepsilon$$

y: data
F: forward problem
ε: noise

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$$y = F(x) + \varepsilon$$

y: data
F: forward problem
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- We cannot do $x = F^{-1}(y)$. We solve the optimization problem

$$x = \underset{x}{argmin} \|y - F(x)\|_2^2$$

The inversion problem

- Find the x-parameters that fulfills

$$y = F(x) + \varepsilon$$

y: data
F: forward problem
ε: noise

- We cannot do $x = F^{-1}(y)$. We solve the optimization problem

$$x = \underset{x}{argmin} \|y - F(x)\|_2^2$$

- Still ill-posed. We introduce some regularization

$$x = \underset{x}{argmin} \{\|y - F(x)\|_2^2 + g(x)\}$$

- Example: sparsity $g(x) = \lambda \|x\|_0$; best subset

The inversion problem

- We want to find the simplest model that reproduce the data
- Evaluating F the minimum amount of times
- With a method that scales linearly with #CPU
- First attempt (to my knowledge) of inversions with a 3D forward solver

The inversion problem

Picture by Rob Glover

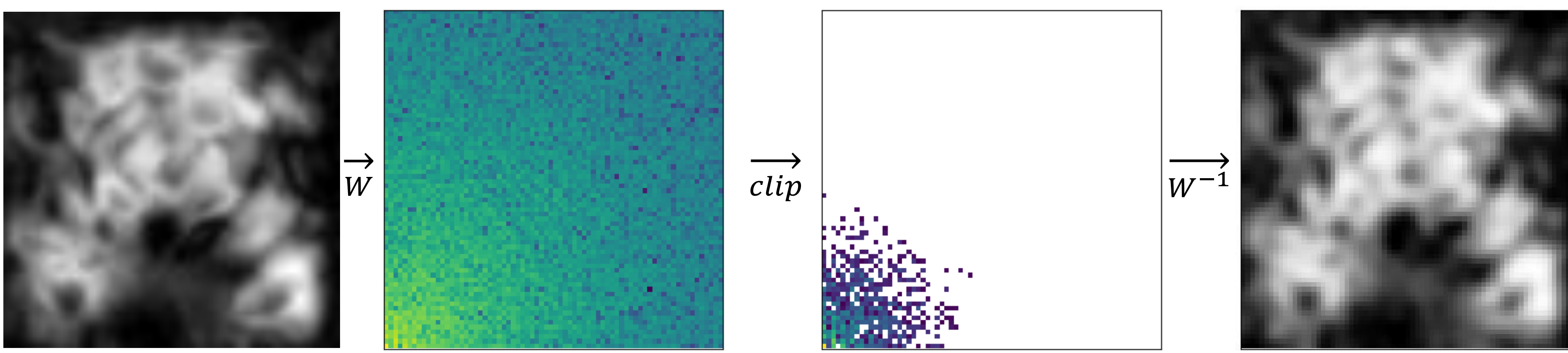
- Sparsity is a rare ocurrence in the *'real'* space

The inversion problem

Picture by Rob Glover

- Sparsity is a rare ocurrence in the *'real'* space

- But can be a common occurrence in a transformed space



The inversion problem

- Find the q-parameters, images of the x-parameters that fulfills

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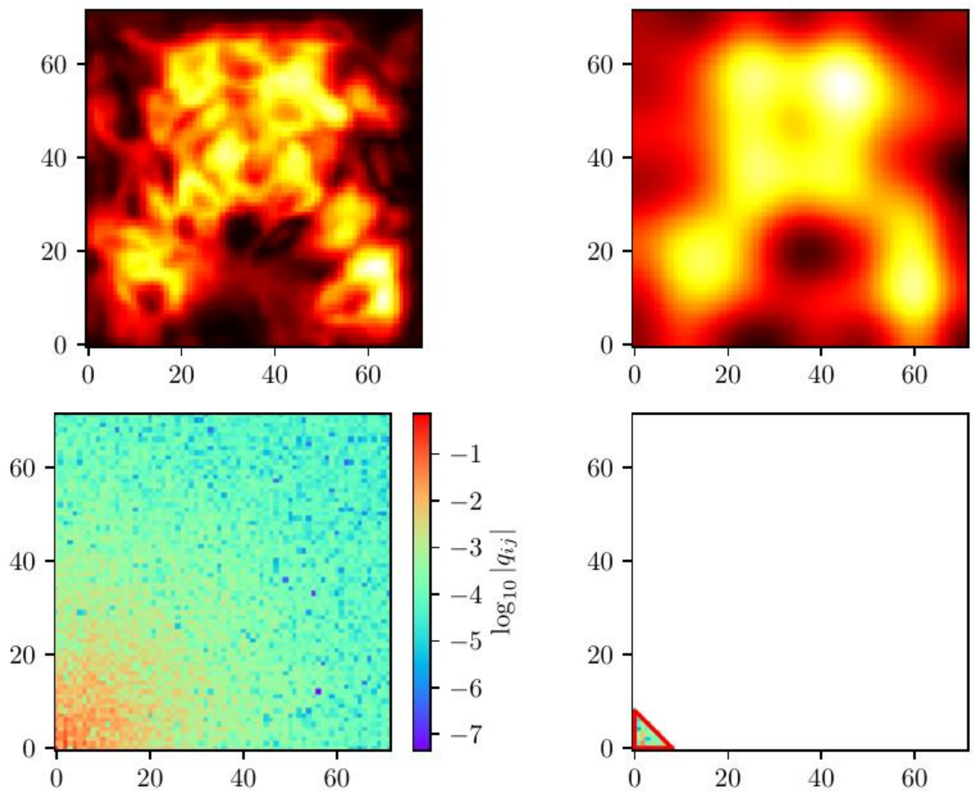
- But we cannot test every q-parameter to check if it is relevant
- Time constrains force us to keep only the smoother modes
- But we loose details

The inversion problem

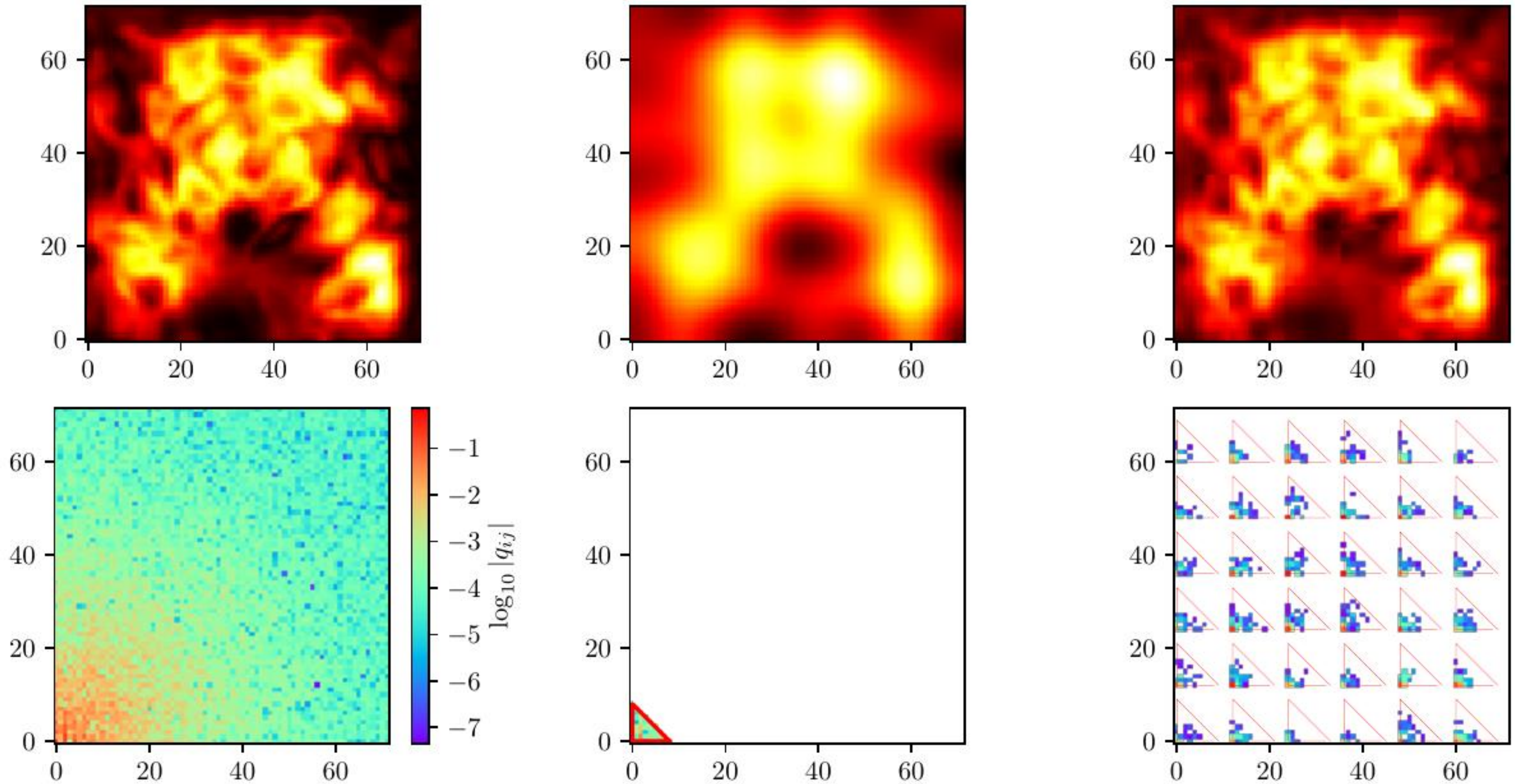


100%

10%

The inversion problem

- We introduce tiling



100% 10% 10%

The inversion problem

- We loose a bit of spatial coherency

The inversion problem

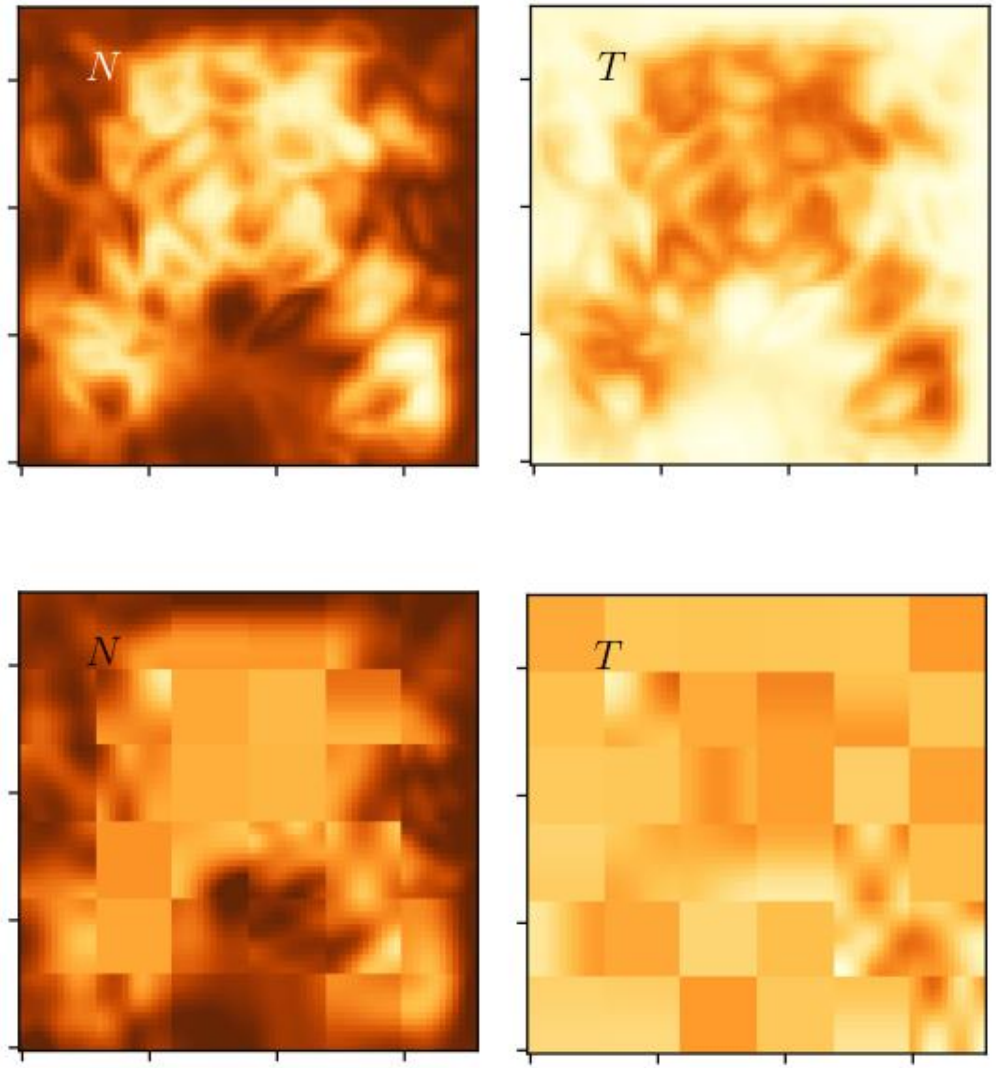
- We loose a bit of spatial coherency
- But everything is still consistent because the tiles interact radiativelly in the forward solver
- We win:
    - More resolution capability
    - Parallel computation of each mode in every tile speed-up by 2-3 orders of magnitude

The inversion problem

- Algorithm:
    - start with very sparse solution
    - iterate until convergence
    - if the agreement not good enough:
      increase modes and repeat

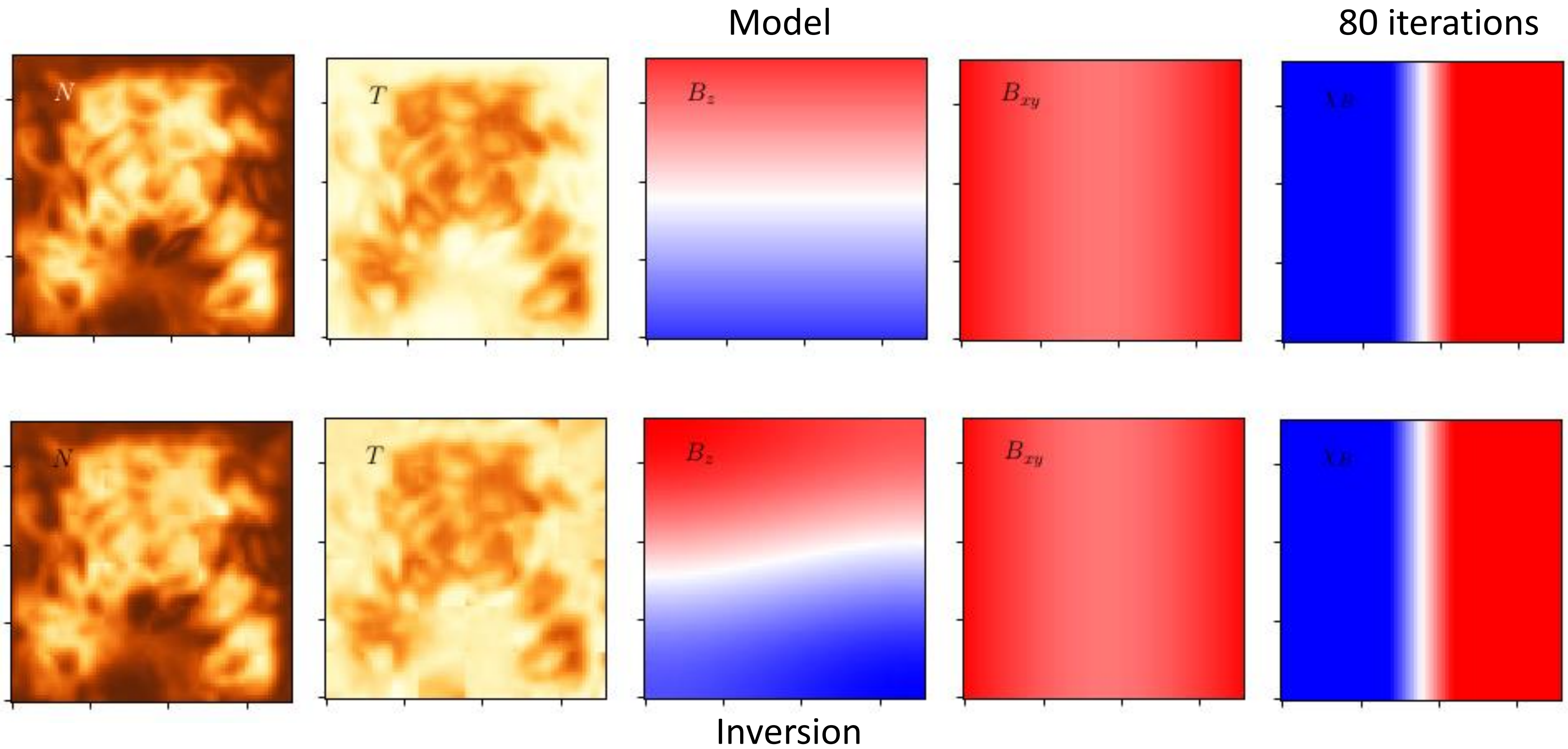
- Result:
    - model with the minimum number of parameters
    - physically consistent

The inversion problem



30 iterations

The inversion problem



The inversion problem

- This is a recent test of feasibility
- We are planing to apply this inversion tool to real data very soon

Thank you for your attention