RES User's Conference, Cáceres. 13-14th September 2022

RED ESPAÑOLA DE SUPERCOMPUTACIÓN

Two-phase Large Eddy Simulation of the jet wiping process at semi-industrial scale

Anne Gosset[1]

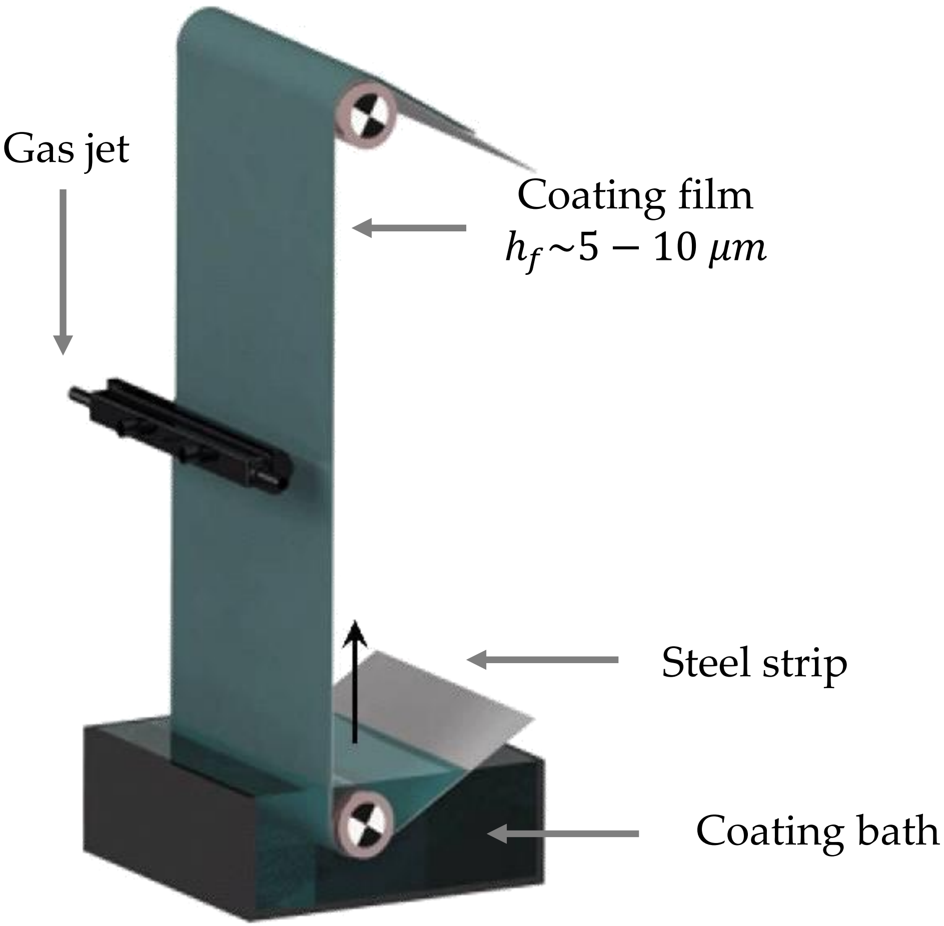
D. Barreiro-Villaverde[2,3], M.A Mendez[3] and M. Lema[2]

[1] CITENI, Campus Industrial de Ferrol
Universidade da Coruña, Spain

[2] CITIC Research **Universidade da Coruña**, Spain
[3] **von Karman Institute**, Sint-Genesius-Rode, Belgium

von KARMAN INSTITUTE FOR FLUID DYNAMICS · UNIVERSIDADE DA CORUÑA · CAMPUSINDUSTRIAL · CITENI · citic

What is jet wiping?

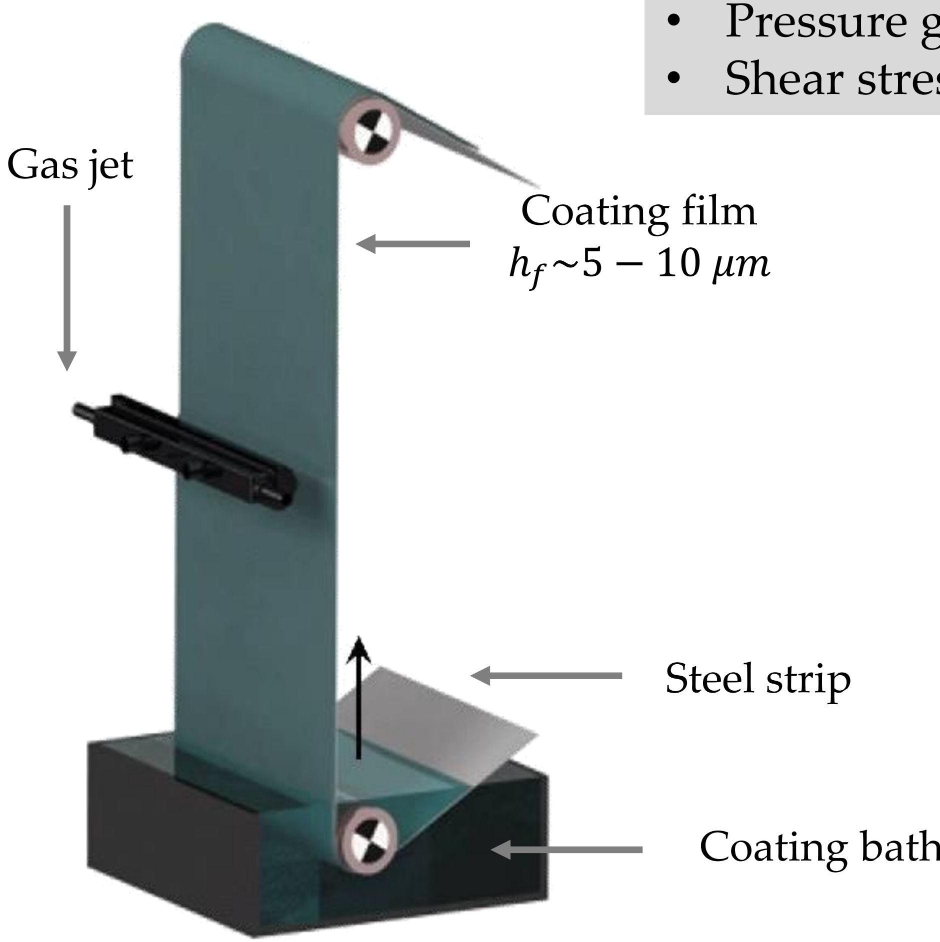


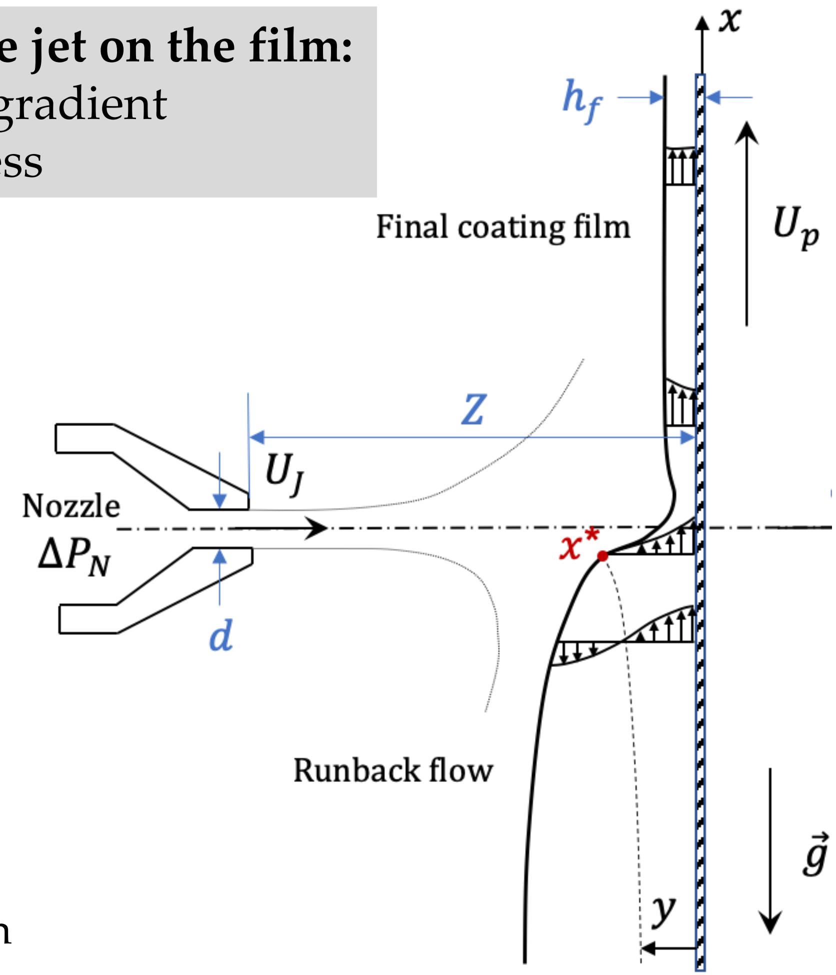
Hot-dip galvanization

550 process lines for over 100 millions tons of galvanized steel per year [ZIC, Cominco ltd]

What is jet wiping?

Action of the jet on the film:
- Pressure gradient
- Shear stress

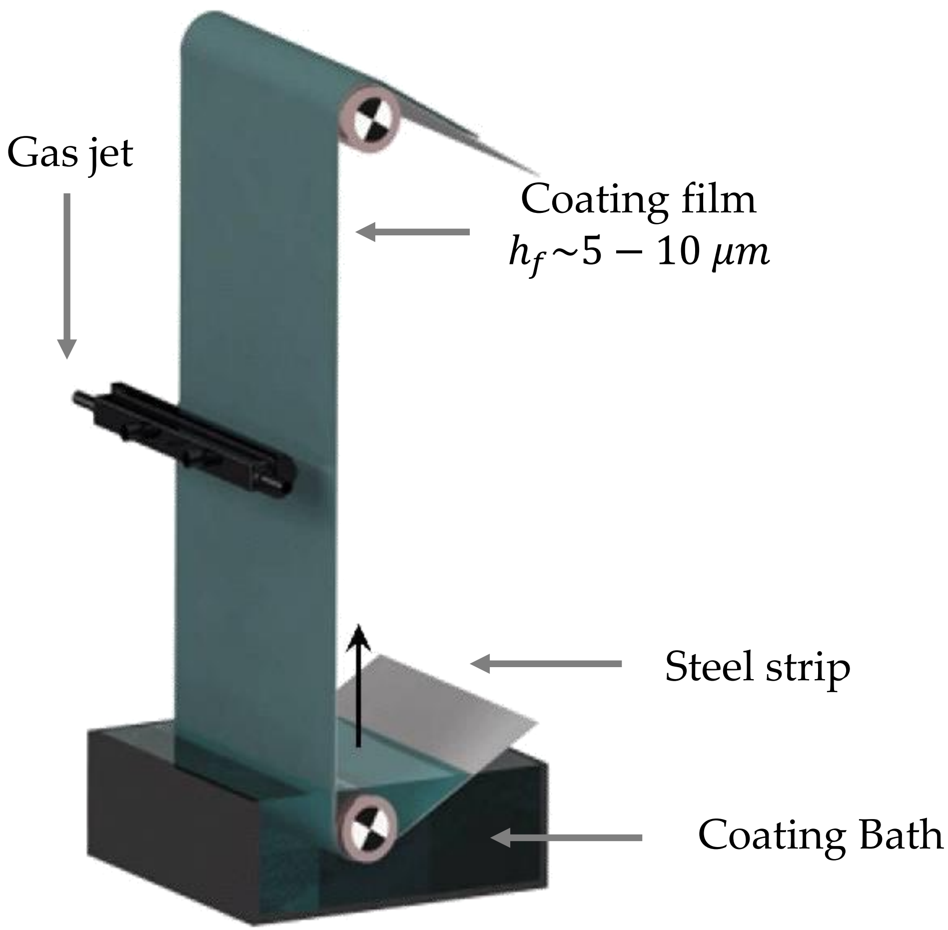




Final coating thickness

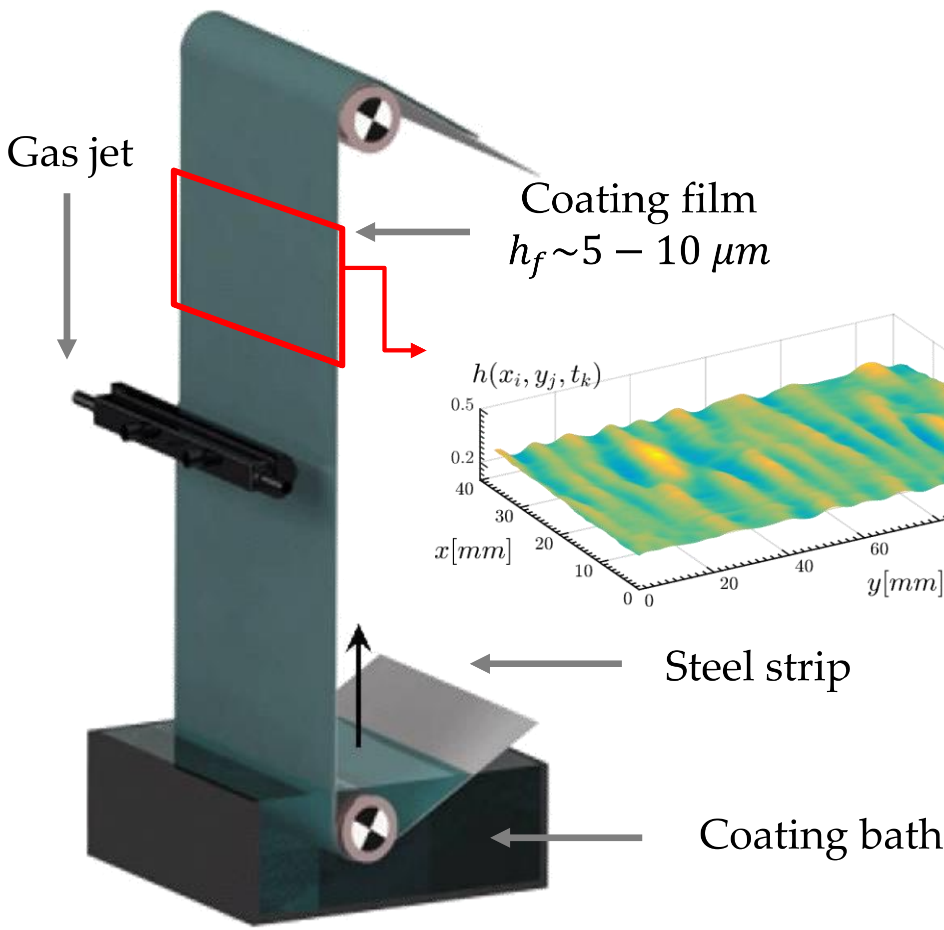
$h_f = f(Z, d, \Delta P_N, U_p, \rho_l, \mu_l)$

What is jet wiping?



- Contactless
- Energy efficient
- Accurate control of process lines

What is jet wiping?



- Contactless
- Energy efficient
- Accurate control of process lines

- Instability on the final zinc film: **Undulation**
- Degradation of the coating quality

The instability mechanism: what we know

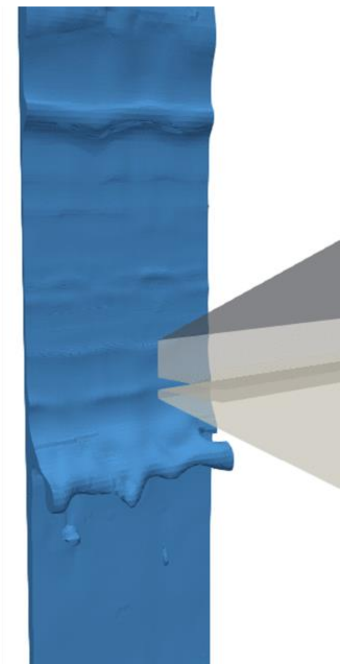
On the dynamics of jet wiping: Numerical simulations and modal analysis

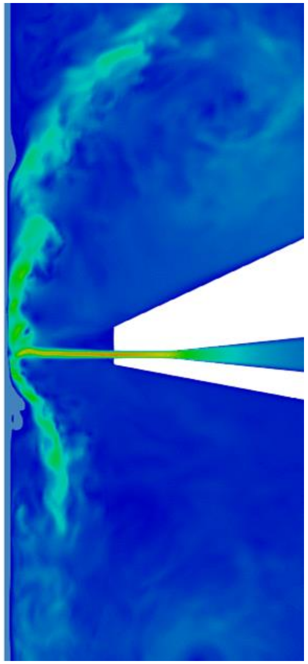
Cite as: Phys. Fluids **33**, 062114 (2021); doi: 10.1063/5.0051451
Submitted: 25 March 2021 · Accepted: 17 May 2021 ·
Published Online: 16 June 2021

View Online

David Barreiro-Villaverde,[1,2,a)] Anne Gosset,[3,b)] and Miguel A. Mendez[2,c)]

RES allocations
FinisTerrae II (FI-2018–3-0040)
Tirant (FI-2019–1-0038)





Combination of:
- High-fidelity CFD simulations
- Multiscale modal analysis

Conclusions:
- Extensive validation of the CFD model with experimental data.
- Description of the mechanism of undulation *in lab-scale conditions*.

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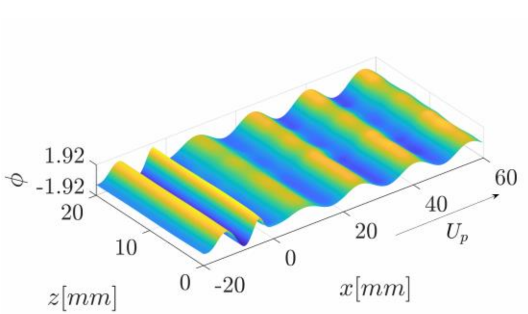

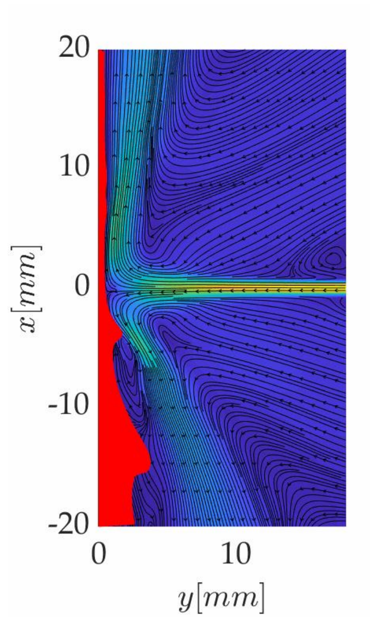

- **2D dominant wave** patterns in both the final film and the runback flow
- Gas flow structures correlated with the coating undulation

→ Robust gas-liquid coupling mechanism

The instability mechanism: what we know

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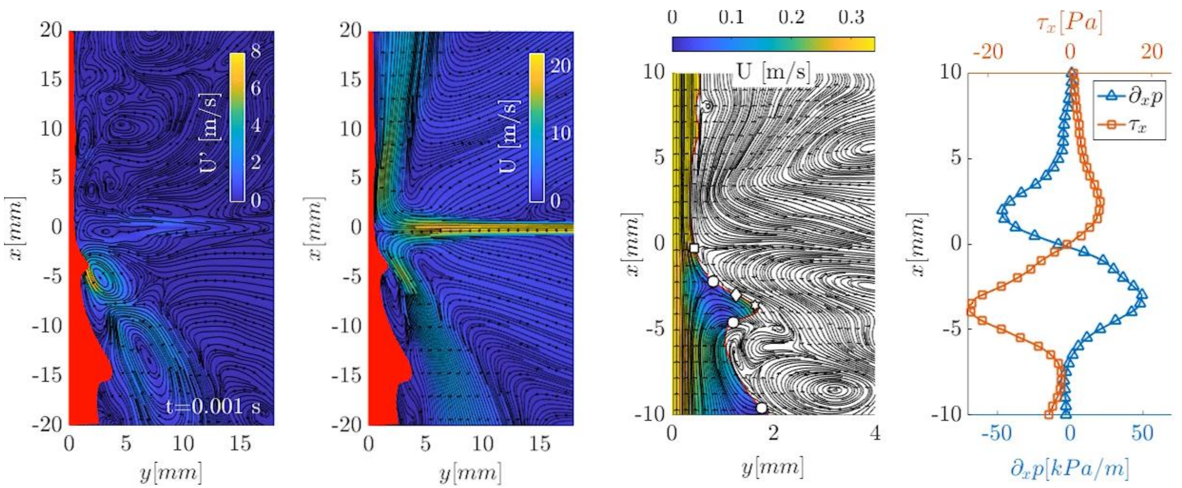


- Correlation of the gas dynamics with the pressure and shear stress distributions at the interface
- Pulsation of the pressure gradient on the runback flow

→ Periodic modification of the wiping efficiency and undulation

Scaling in jet wiping

Relevant dimensionless numbers for similarity

Liquid

$$Re_f = \frac{U_p h_f}{\nu_l} \qquad Re_l = \frac{U_p [h]}{\nu_l}$$

$$Ca = \frac{\mu_l U_p}{\sigma} \qquad \epsilon = \frac{[h]}{[x]} = Ca^{1/3}$$

$$Ka = \frac{\sigma}{\rho_l \, g^{1/3} \nu^{4/3}}$$

Gas

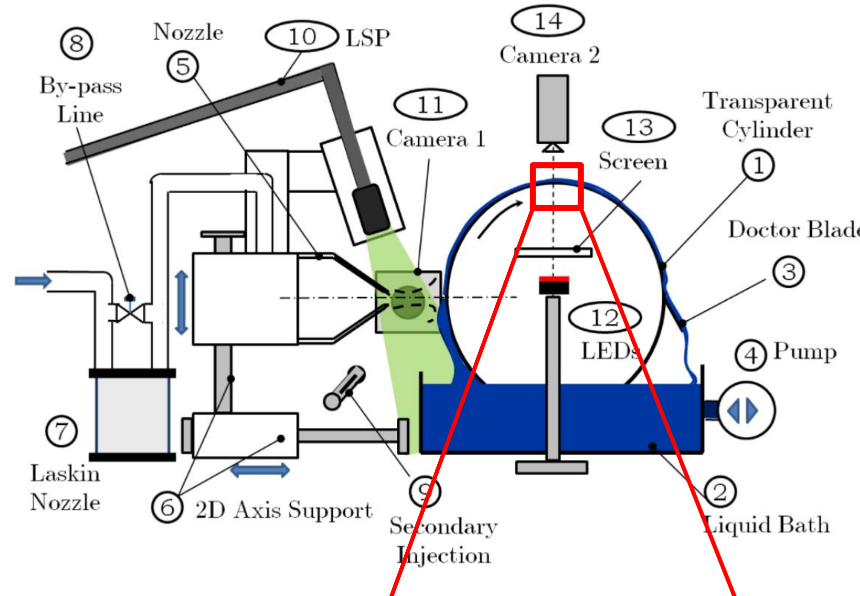
$$Re_j = \frac{U_j d}{\nu_g} \qquad \widehat{\nabla P}_{max} = \frac{\nabla P_{max}[x]}{[P]}$$

$$\Pi_g = \frac{\Delta P_N d}{\rho_l g Z^2} \qquad \hat{\tau}_{max} = \frac{\tau_{max}}{[\tau]}$$

[1] Gosset, Mendez & Buchlin
Exp. Th. Fluid Sci. 103 (2019).

Lab-scale conditions

- Dipropylene glycol
- Transitional jet flow



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Lab-scale conditions

- Dipropylene glycol
- Transitional jet flow

	[1] Dipropylene		Zinc *
Re_l	7.4		478
Re_f	2.1		18
Ca	0.80	$\neq$	0.0037
Ka	4.8		16440
ϵ	0.93		0.15
Re_j	2380		14370
Π_g	0.16		2.45
$\hat{\nabla} p_g$	5.6		87.6
$\hat{\tau}_g$	0.3		7.2

*$\Delta P_N = 20\ kPa \mid U_p = 1\ m/s \mid Z = 12\ mm \mid d = 1.2\ mm$

[3] Barreiro-Villaverde, Gosset & Mendez Phys. Fluids. 33 (2021).

Scaling in jet wiping

Relevant dimensionless numbers for similarity

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Wiping with water

	[1] Dipropylene		Water		Zinc*
Re_l	7.4		320		478
Re_f	2.1		28		18
Ca	0.80	$\neq$	0.014	$\approx$	0.0037
Ka	4.8		3400		16440
ϵ	0.93		0.24		0.15
Re_j	2380		2830		14370
Π_g	0.16		1.02		2.45
$\hat{\nabla} p_g$	5.6		36.4		87.6
$\hat{\tau}_g$	0.3		3.7		7.2

Air-water wiping configurations:

Case #	U_p [m/s]	ΔP_N [Pa]	Π_g [-]	Re_f
1	1	1000	1.02	28
2	1.5	1000	1.02	54
3	1	1500	1.53	22

$^*\Delta P_N = 20\ kPa \mid U_p = 1 \mid Z = 12\ mm \mid d = 1.2\ mm$

Research Questions

1) What is the mechanism responsible for undulation in improved similarity conditions?

2) Is it possible to simulate jet wiping using simplified models for the liquid film?

3D high-fidelity simulations

Modal analysis

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3D high-fidelity simulations

Integral model for the liquid film

Research Questions

1) **What is the mechanism responsible for undulation in improved similarity conditions?**

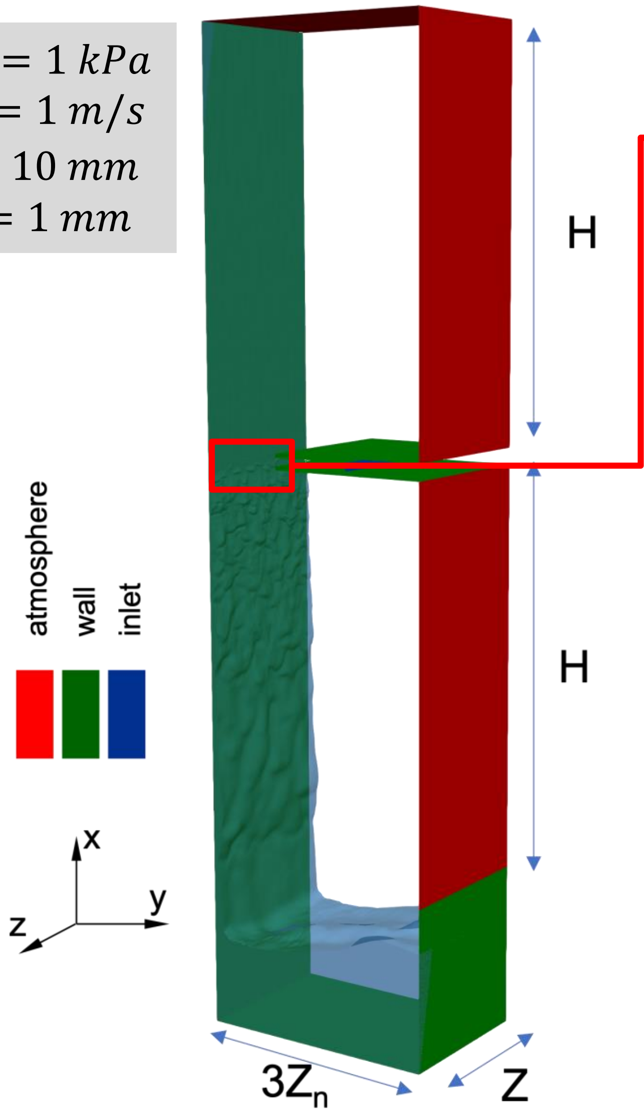
2) Is it possible to simulate jet wiping using simplified models for the liquid film?

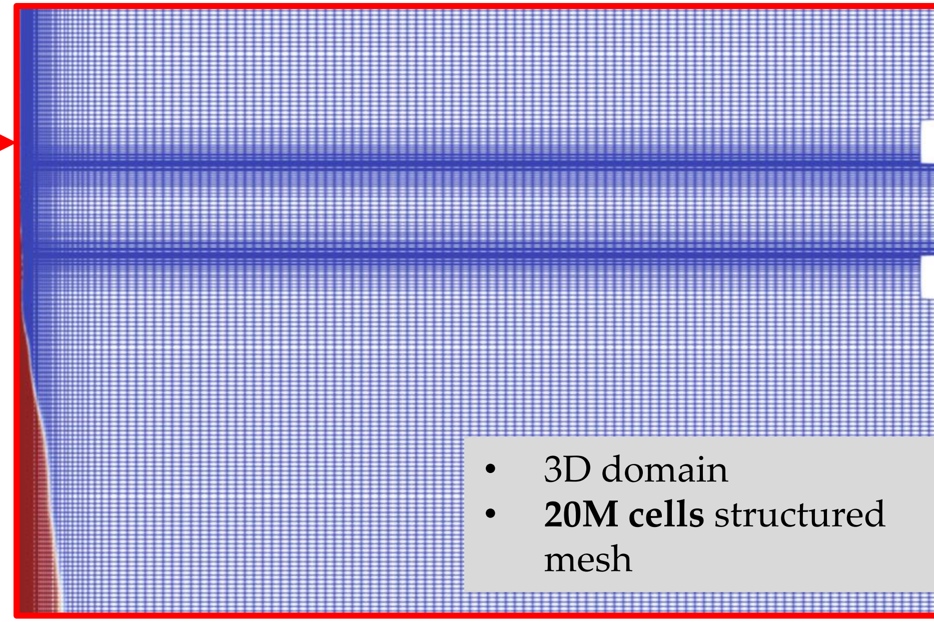
3D high-fidelity Simulations

Modal analysis

Test case and numerical methodology

$\Delta P_N = 1\ kPa$
$U_p = 1\ m/s$
$Z = 10\ mm$
$d = 1\ mm$





- 3D domain
- **20M cells** structured mesh

Water
Density: $\rho_l = 1000\ kg/m^3$
Viscosity: $\mu_l = 0{,}001\ Pa \cdot s$
Surface tension: $\sigma_l = 0{,}072\ N/m$

Test case and numerical methodology

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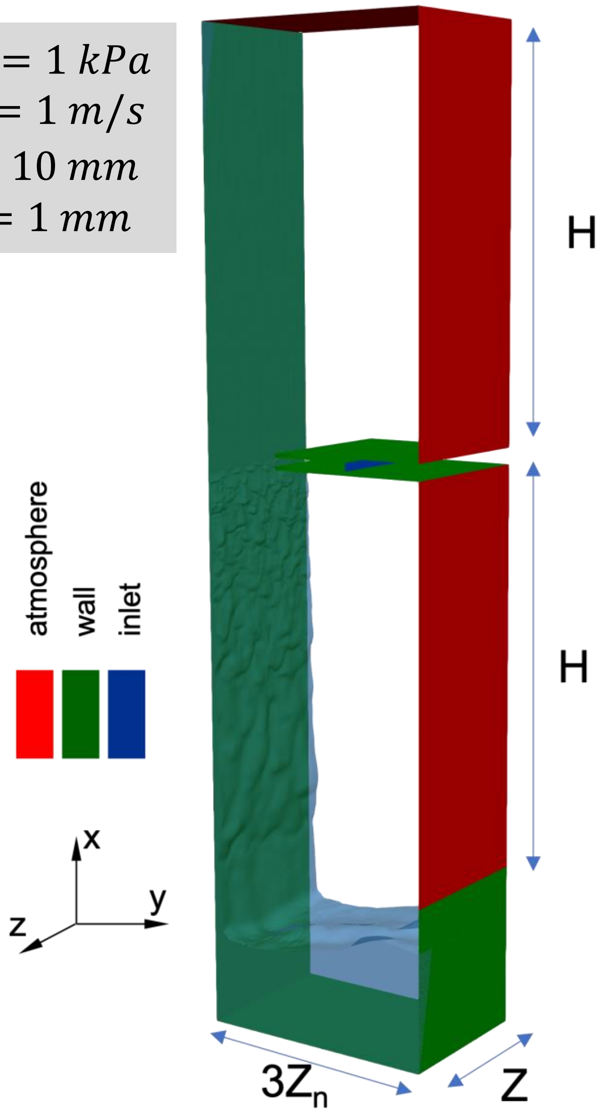
$U_p = 1\ m/s$
$Z = 10\ mm$
$d = 1\ mm$



Numerical configuration

- Finite Volume Libraries OpenFOAM® v9
- Two-phase solver: interFoam (**Volume of Fluid**)
- Smagorinski **LES** (Large Eddy Simulation)

⟶ Largest scales of turbulence are fully resolved.

Computational cost

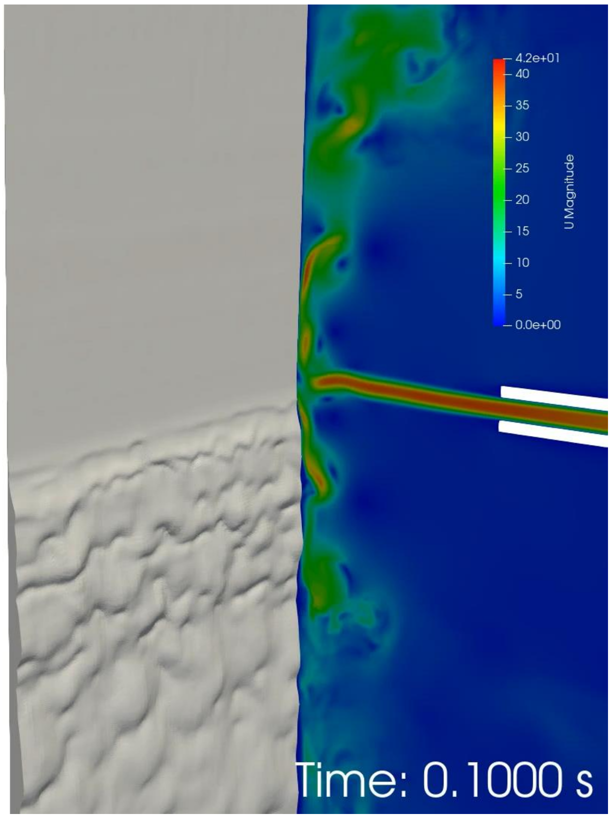
- **2700 - 4300 hours CPU/s** of real flow using 1024 processors with water.
- **10M CPU** hours of priority access to the **Spanish Supercomputing Network (RES)**.
- **>10 TB** of data.

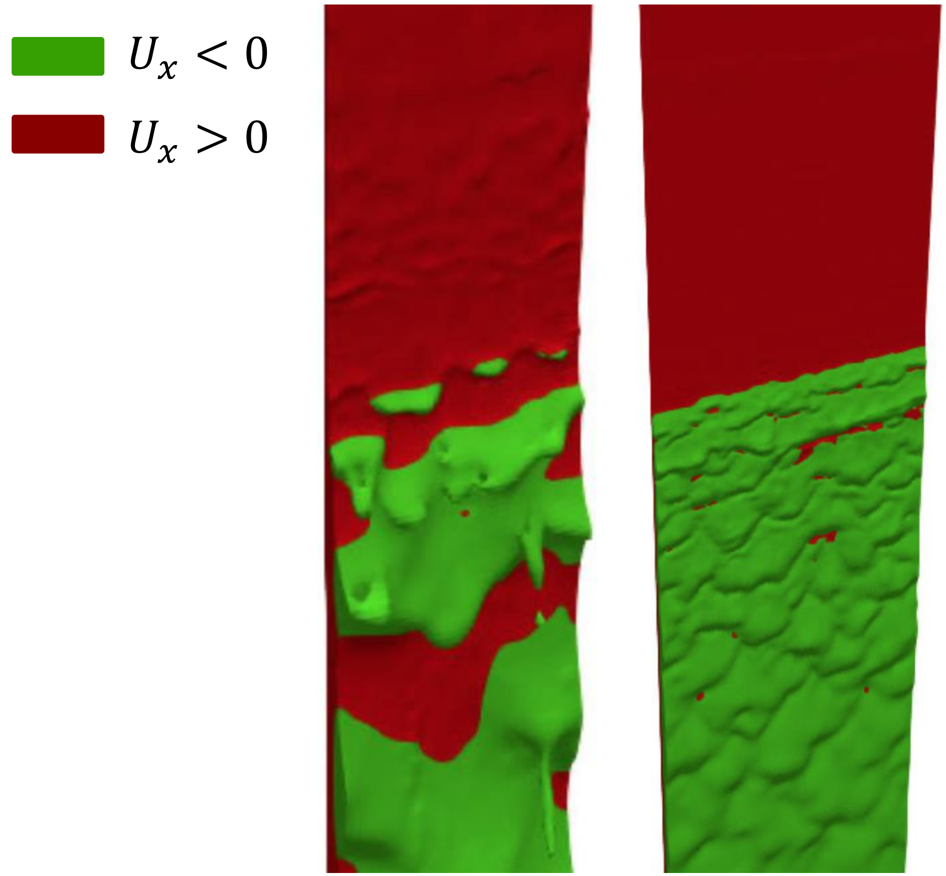
CESGA
Centro de Supercomputación de Galicia

Mechanism of undulation in improved similarity

The undulation mechanism with water

$\Delta P_N = 1000\ Pa$
$U_p = 1\ m/s$





Dipropylene
$\Delta P_N = 875\ Pa$
$U_p = 0.34\ m/s$

Water
$\Delta P_N = 1500\ Pa$
$U_p = 1\ m/s$

RES allocation
FinisTerrae II and III (FI-2021–3-0012)

Post-processing with modal analysis

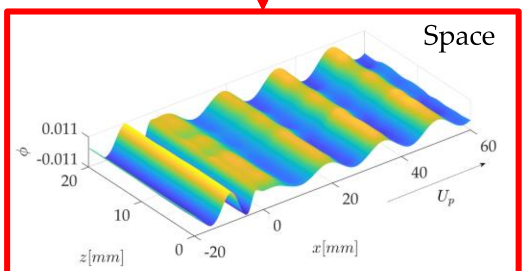
Post-processing using modal analysis

Modal analysis is used in fluid mechanics with different objectives:

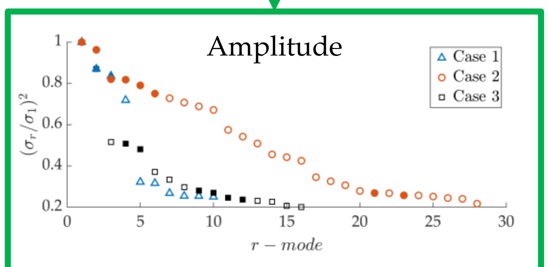
- Reduced Order Models (ROM).
- Data compression/**filtering**.
- **Pattern recognition.**

$$D[i,k] = \begin{array}{c|c} & \xrightarrow{\text{time}} \\ \text{space}\downarrow & \begin{bmatrix} d_1[1] & \cdots & d_{n_t}[1] \\ \vdots & \ddots & \vdots \\ d_1[n_s] & \cdots & d_{n_t}[n_s] \end{bmatrix} \end{array}$$

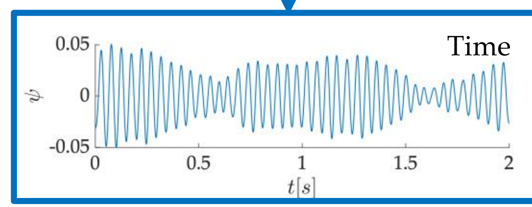
$$\tilde{D}[i,k] = \underbrace{\sum_{r=1}^{r_c<rank(D)}}_{\text{r modes}} \boxed{\phi_r}\,\boxed{\Sigma_r}\,\boxed{\psi_r^T}$$



How the energy is distributed in space



How dominant is each mode compared to the others

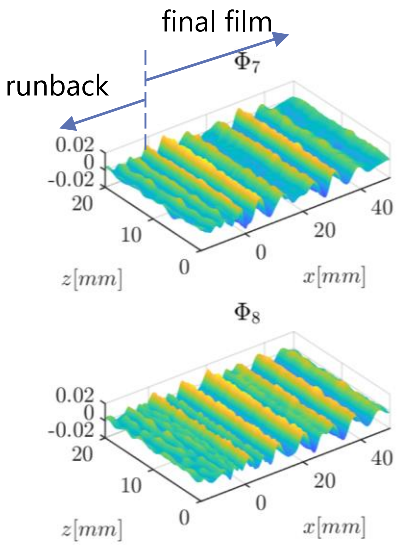


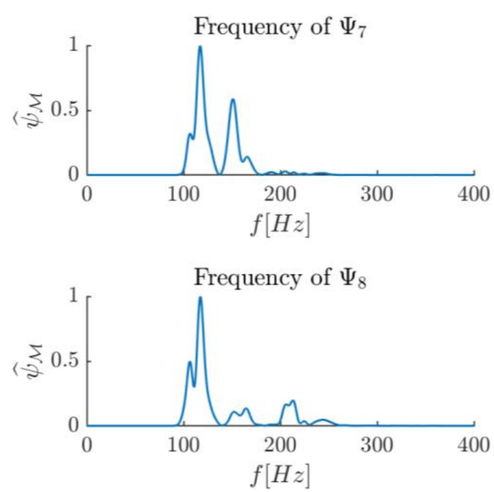
How the structure evolves in time

[4] Mendez, Balabane & Buchlin
J. Fluid. Mech. 870 (2019).

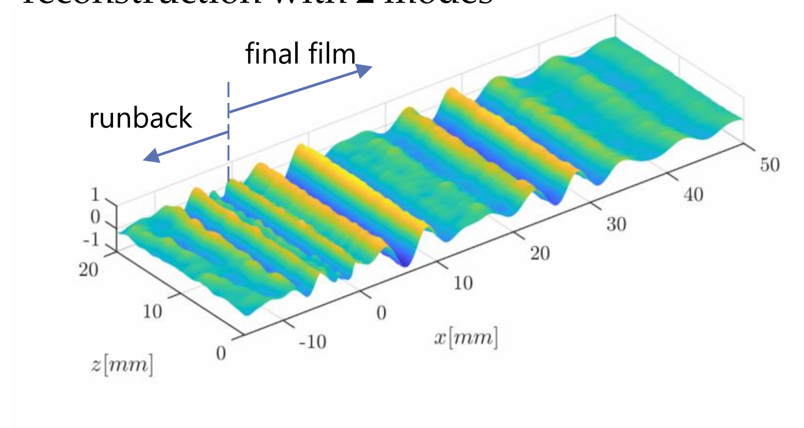
Mechanism of undulation in improved similarity

The undulation mechanism with water:
Liquid film





Normalized thickness reconstruction with 2 modes



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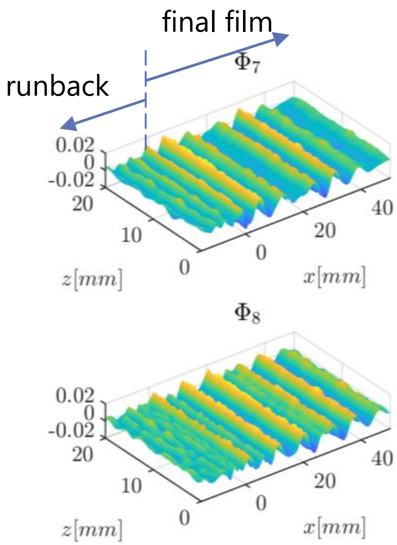

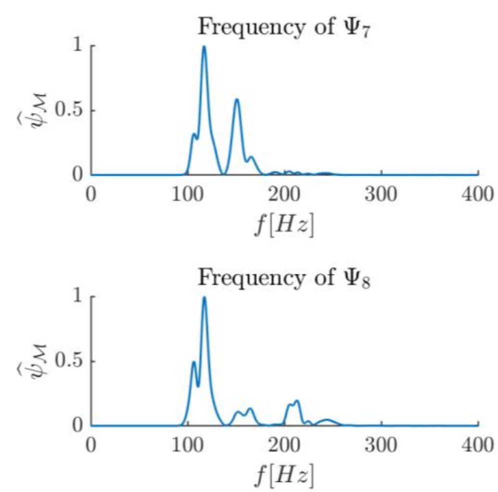

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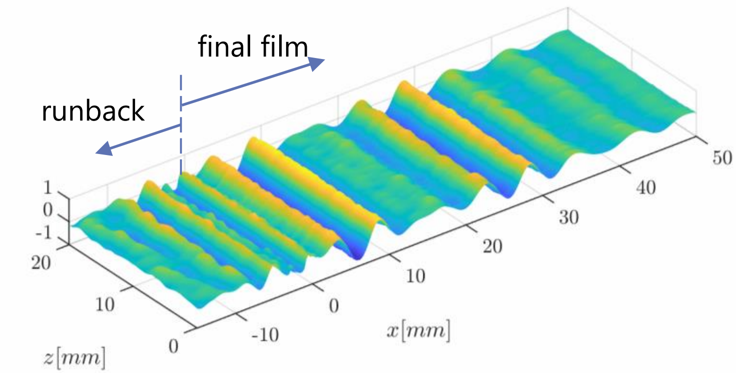
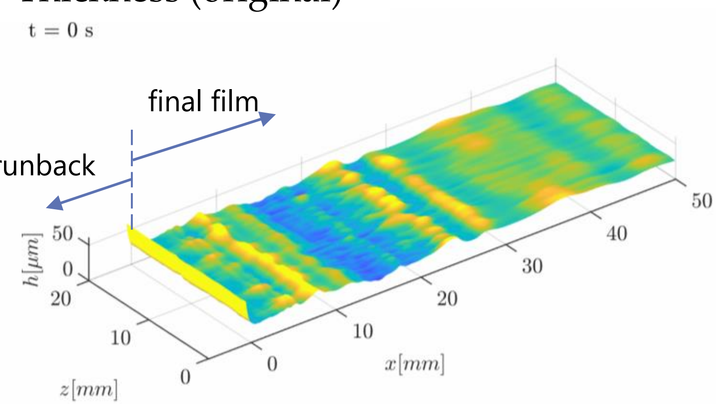
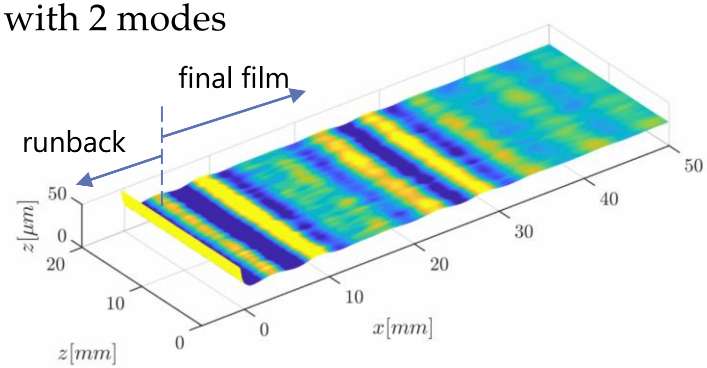


Thickness (original)



Thickness reconstruction with 2 modes

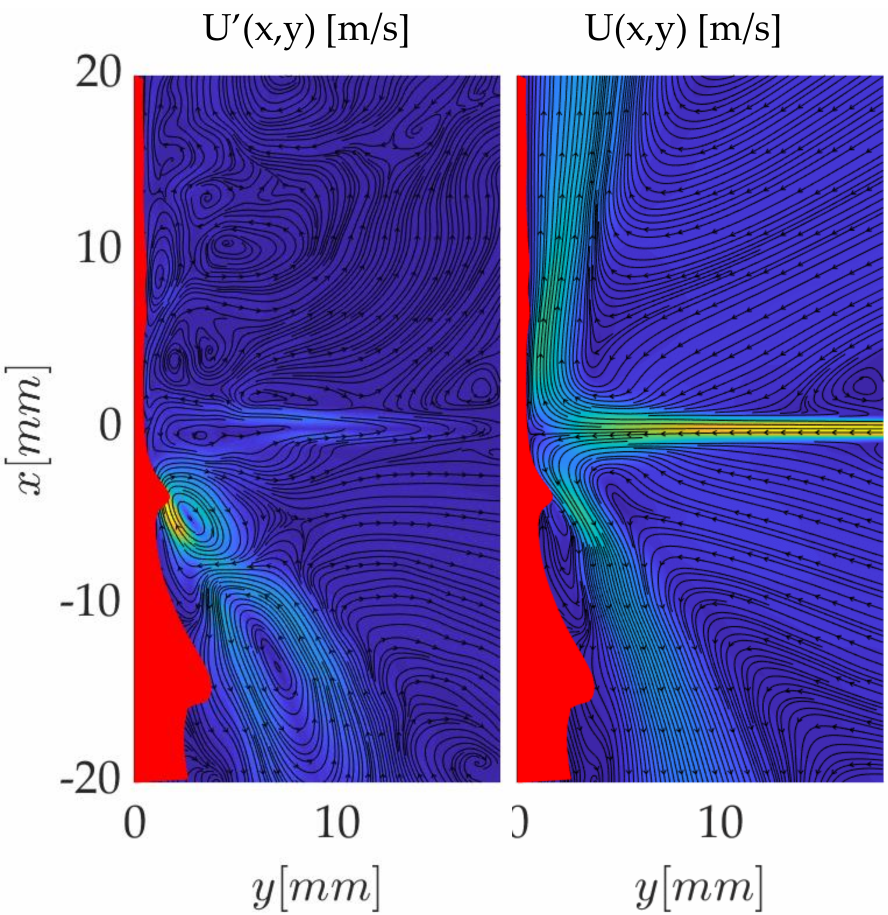


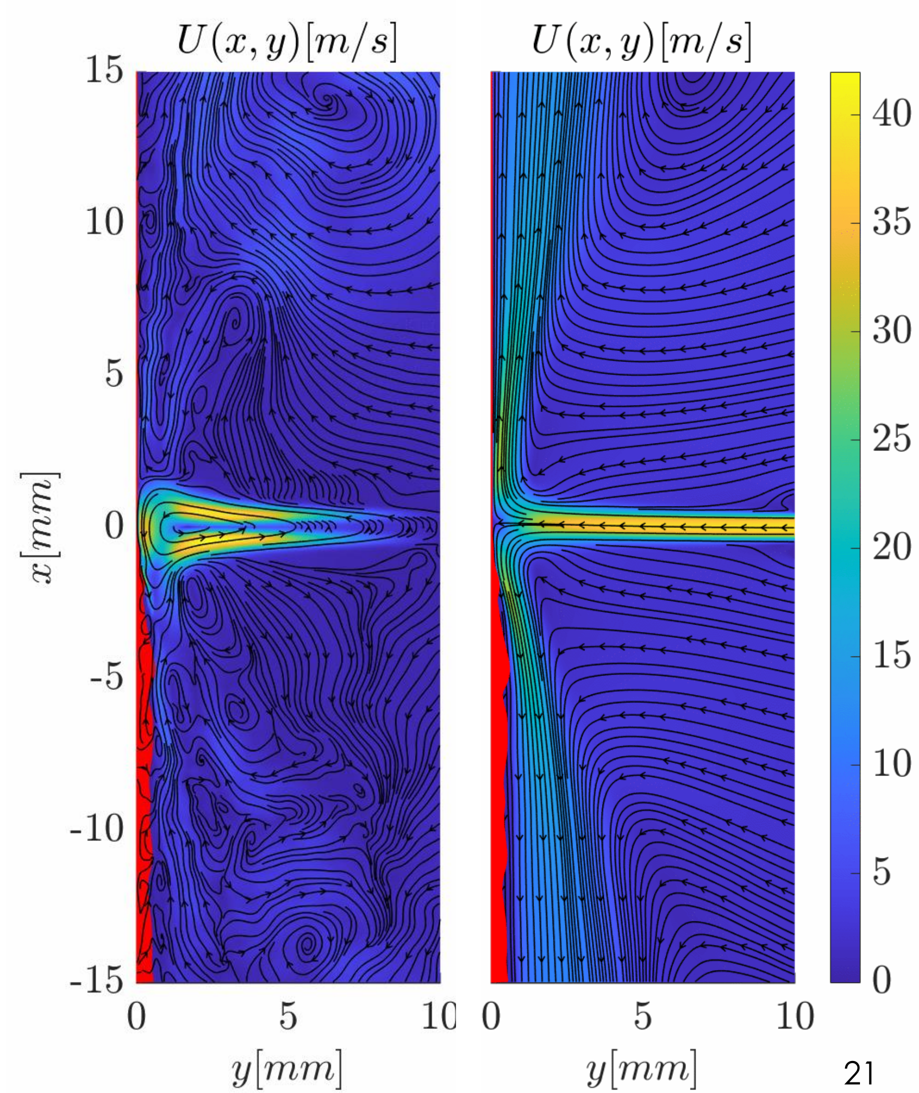
Mechanism of undulation in improved similarity

The undulation mechanism with water:
Gas jet

Water

Dipropylene glycol





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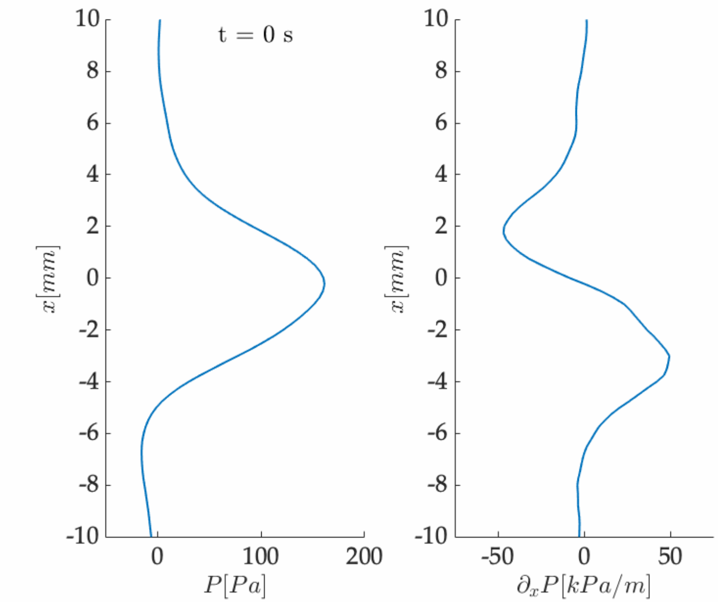
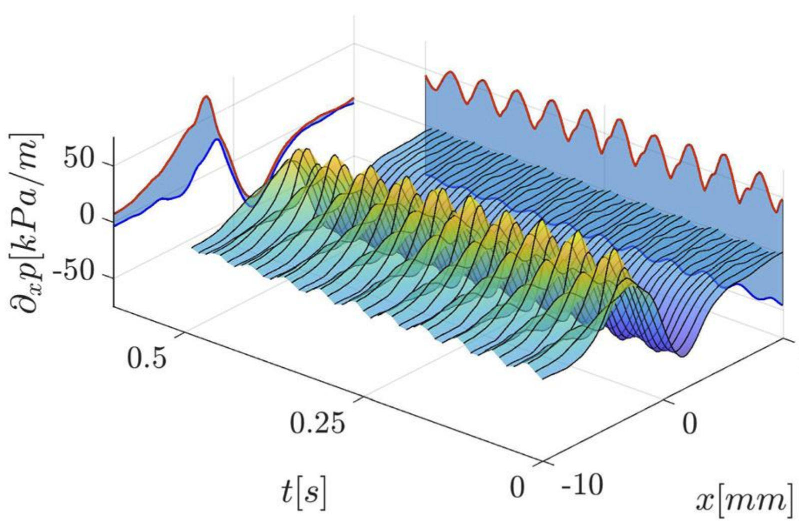
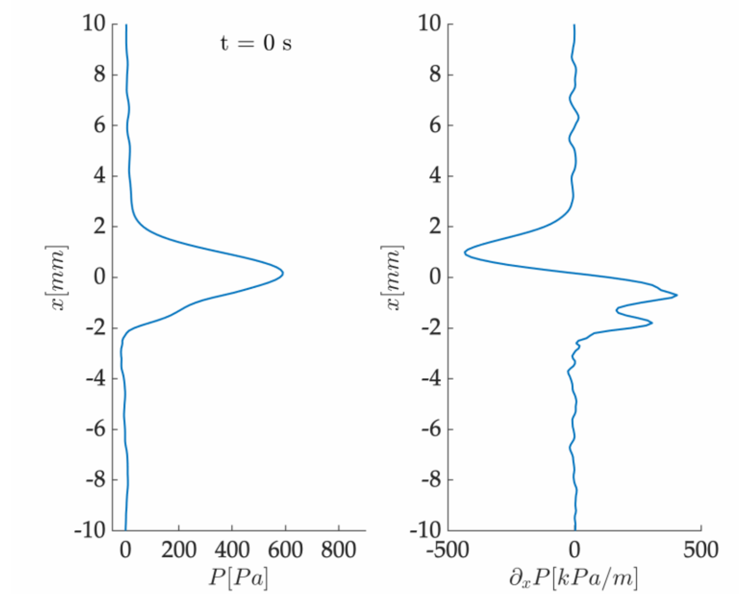
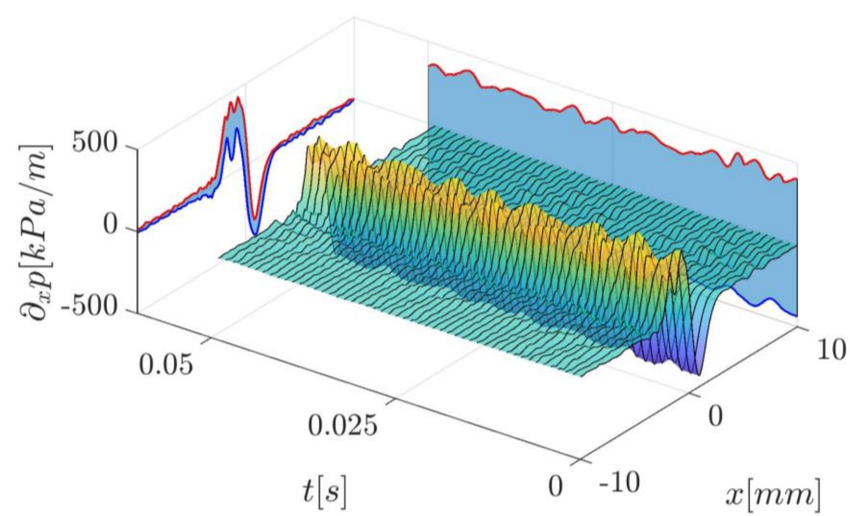




Water





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3D high-fidelity simulations

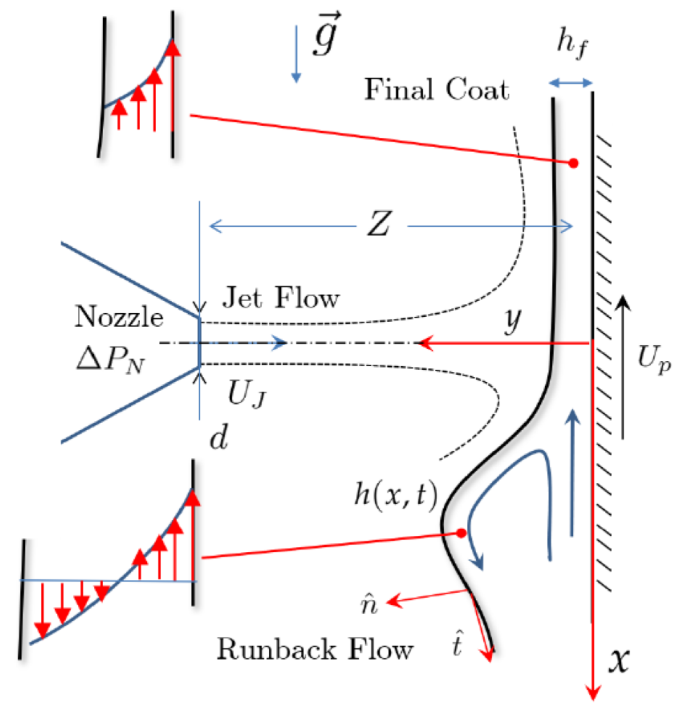
Integral model for the liquid film

Validation of integral models for jet wiping

The integral boundary layer model (**BLEW**) [5] developed in the von Karman Institute is based on the **long-wavelength approximation** ($\epsilon = [h]/[x] \ll 1$).

$$\partial_{\hat{t}}\hat{h} = -\partial_{\hat{x}}\hat{q}$$

$$\partial_{\hat{t}}\hat{q} = -\partial_{\hat{x}}\hat{\mathcal{F}}(\hat{q},\hat{h}) - \hat{h}\partial_{\hat{x}}\hat{p}_g + \hat{h} + \frac{\epsilon^3}{Ca}\hat{h}\partial_{\hat{x}\hat{x}\hat{x}}\hat{h} + \hat{\tau}_g + \hat{\tau}_w(\hat{q},\hat{h})$$



[5] Mendez, Gosset, Scheid, Balabane & Buchlin J. Fluid. Mech. 911, A47 (2021).

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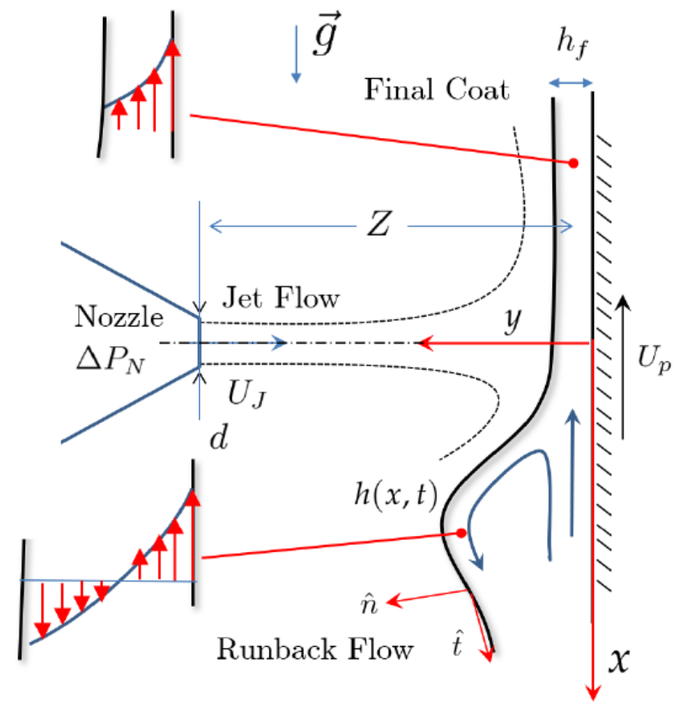
$$\partial_{\hat{t}} \hat{h} = -\partial_{\hat{x}} \hat{q}$$

Only 2 unknowns!

$$\partial_{\hat{t}} \hat{q} = -\partial_{\hat{x}} \hat{\mathcal{F}}(\hat{q}, \hat{h}) - \hat{h} \partial_{\hat{x}} \hat{p}_g + \hat{h} + \frac{\epsilon^3}{Ca} \hat{h} \partial_{\hat{x}\hat{x}\hat{x}} \hat{h} + \hat{\tau}_g + \hat{\tau}_w(\hat{q}, \hat{h})$$



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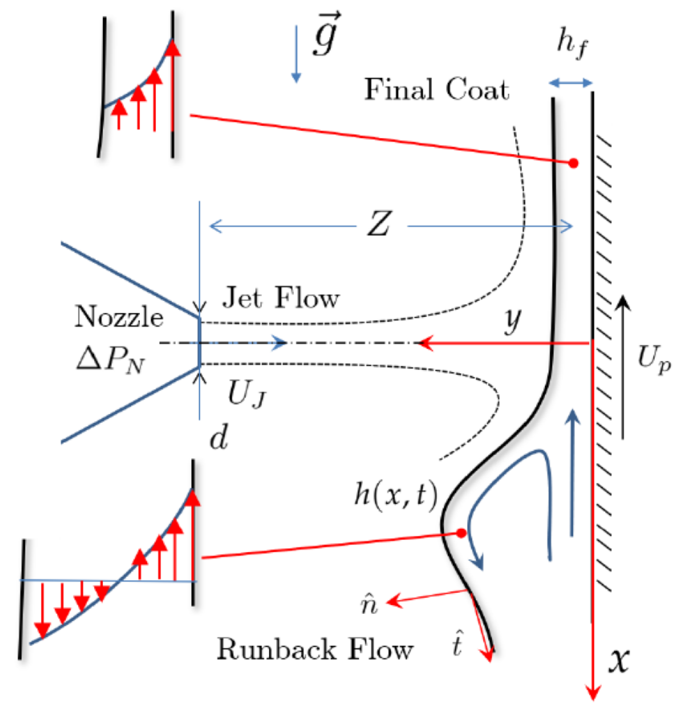
2 inputs: Effect of the jet on the liquid film



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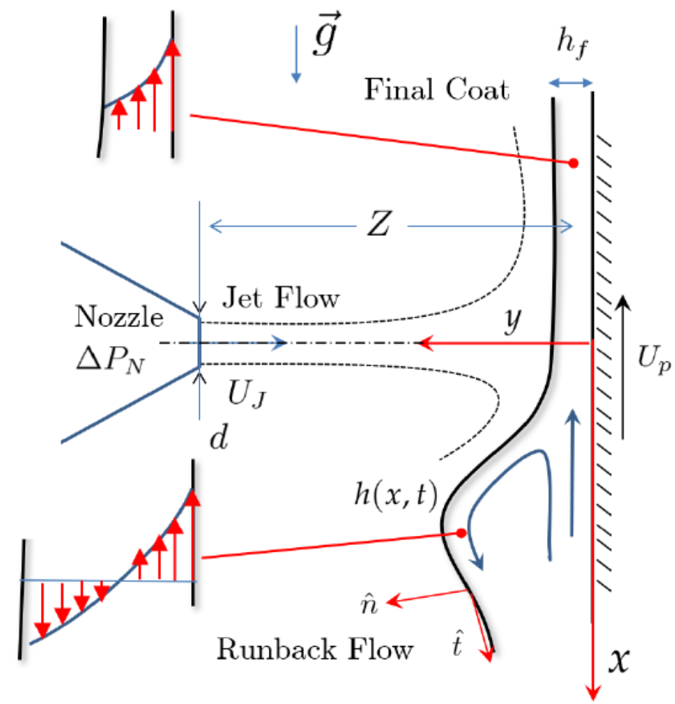
Assumptions:
- Long-wavelength
- Self-similarity of the velocity profiles.
- **One-way coupling: the jet is not affected by the film dynamics.**



[5] Mendez, Gosset, Scheid, Balabane & Buchlin J. Fluid. Mech. 911, A47 (2021).

Validation of integral models for jet wiping

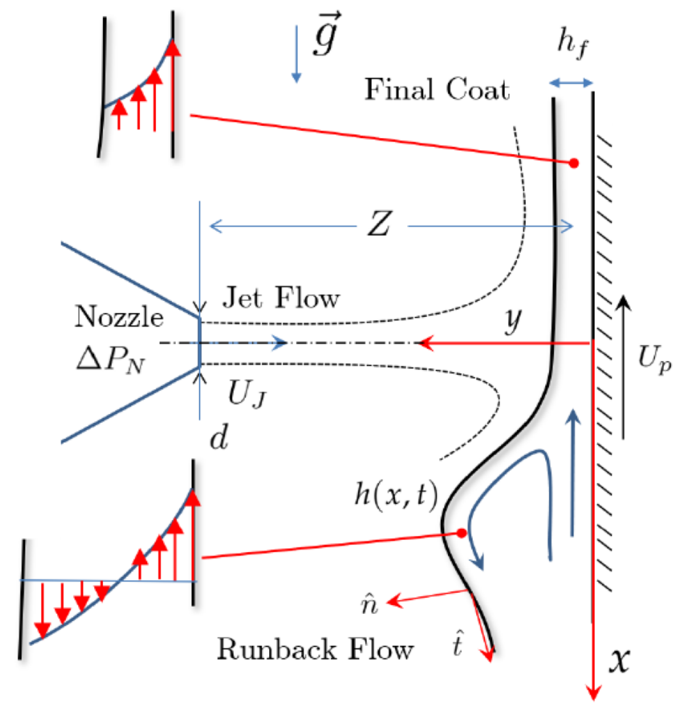
The integral boundary layer model (**BLEW**) [5] developed at the von Karman Institute is based on the **long-wavelength approximation** ($\epsilon = [h]/[x] \ll 1$).

$$\partial_{\hat{t}}\hat{h} = -\partial_{\hat{x}}\hat{q}$$

$$\partial_{\hat{t}}\hat{q} = -\partial_{\hat{x}}\hat{\mathcal{F}}(\hat{q},\hat{h}) - \hat{h}\partial_{\hat{x}}\hat{p}_g + \hat{h} + \frac{\epsilon^3}{Ca}\hat{h}\partial_{\hat{x}\hat{x}\hat{x}}\hat{h} + \hat{\tau}_g + \hat{\tau}_w(\hat{q},\hat{h})$$

Assumptions:

- Long-wavelength
- Self-similarity of the velocity profiles.
- **One-way coupling: the jet is not affected by the film dynamics.**



[5] Mendez, Gosset, Scheid, Balabane & Buchlin J. Fluid. Mech. 911, A47 (2021).

✓ Low computational cost

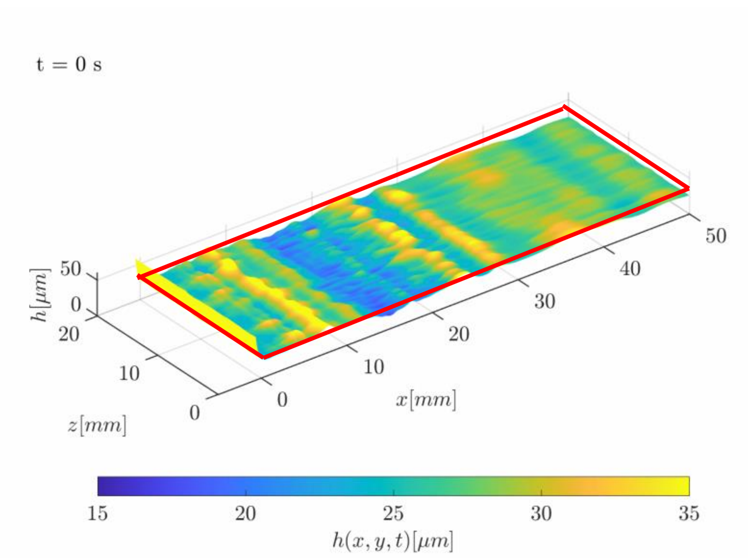
✗ One-way coupling

Validation of integral models for jet wiping

The validation is based on the comparison of the final film dynamics using:

- **CFD simulations**

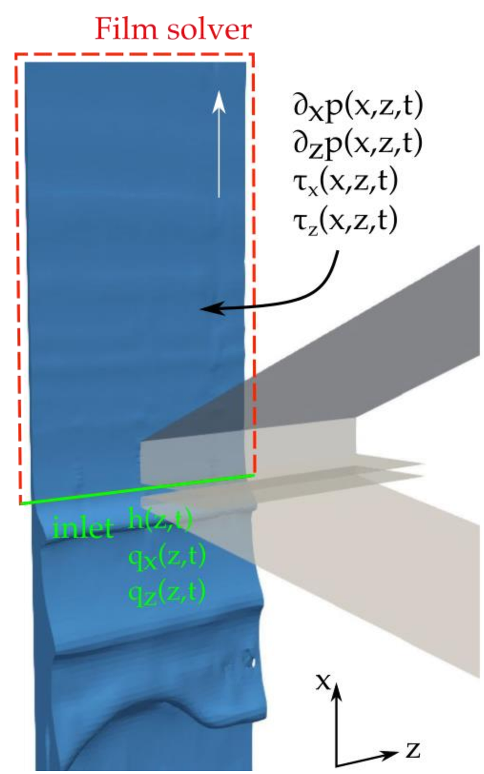
The volume fraction fields α are post-processed to retrieve the spatio-temporal thickness distributions.



- **Integral model + CFD data**

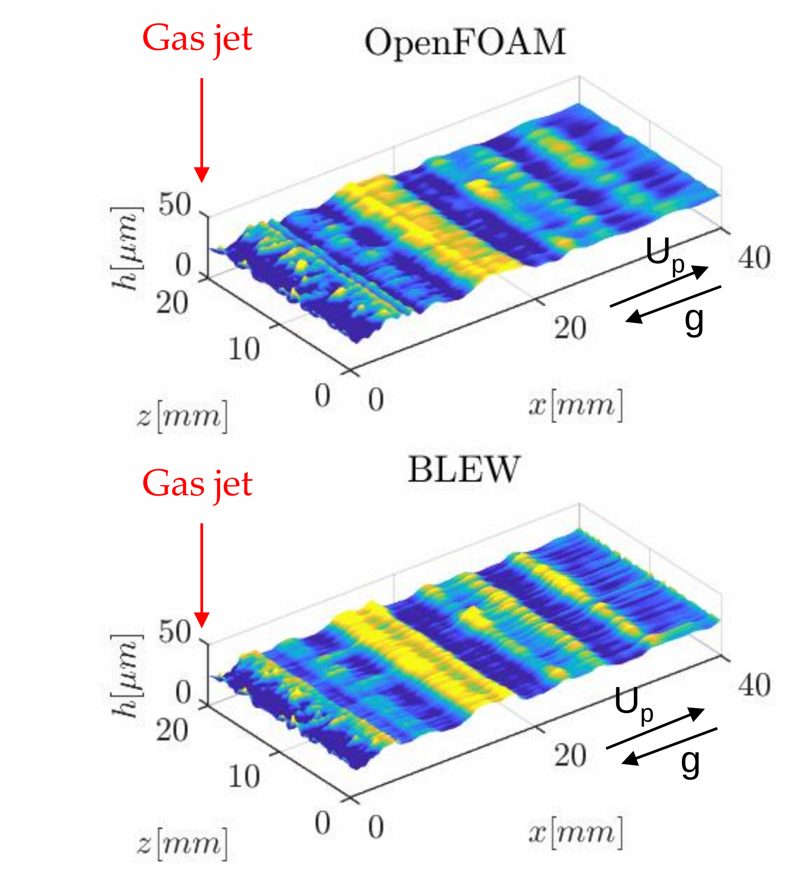
The integral model is fed with:

- $h(z,t), q_x(z,t), q_z\ (z,t)$ at impingement (x=0).
- Spatiotemporal pressure gradient and shear stress

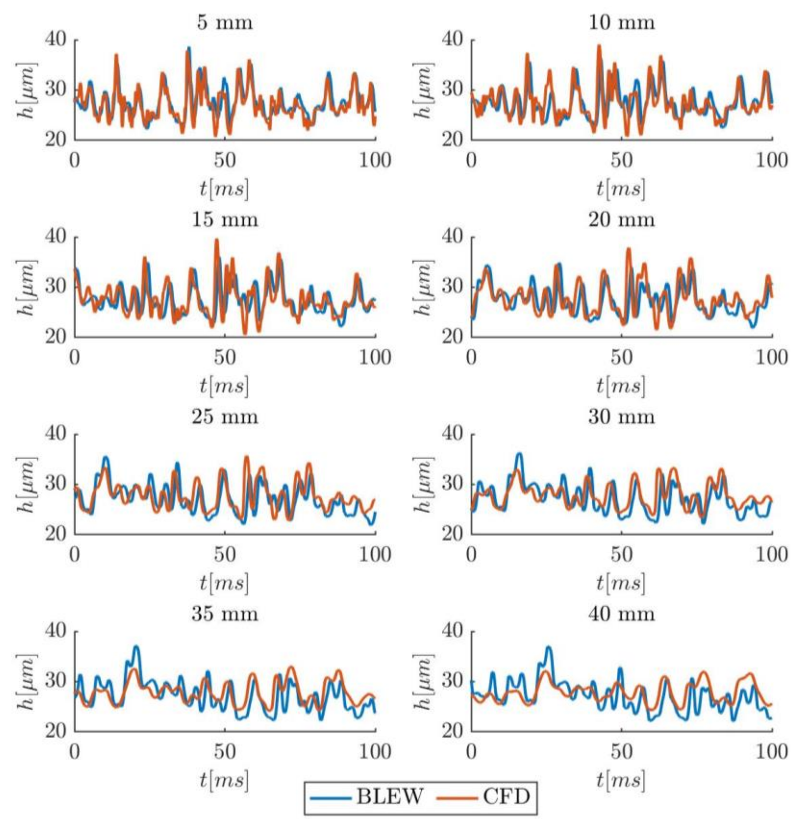


Validation of integral models for jet wiping

Qualitative comparison

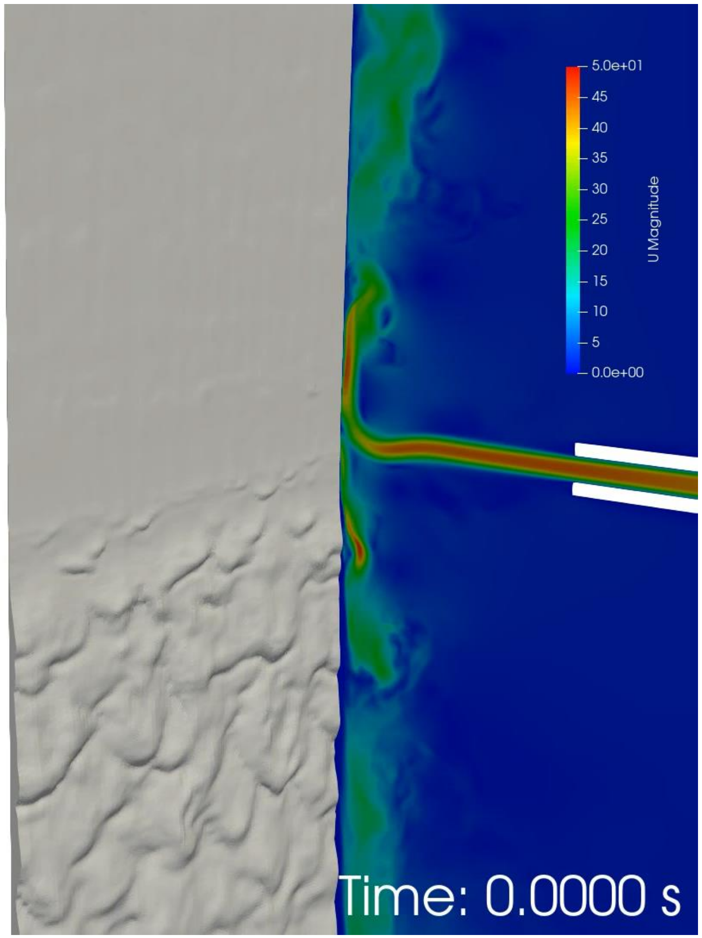


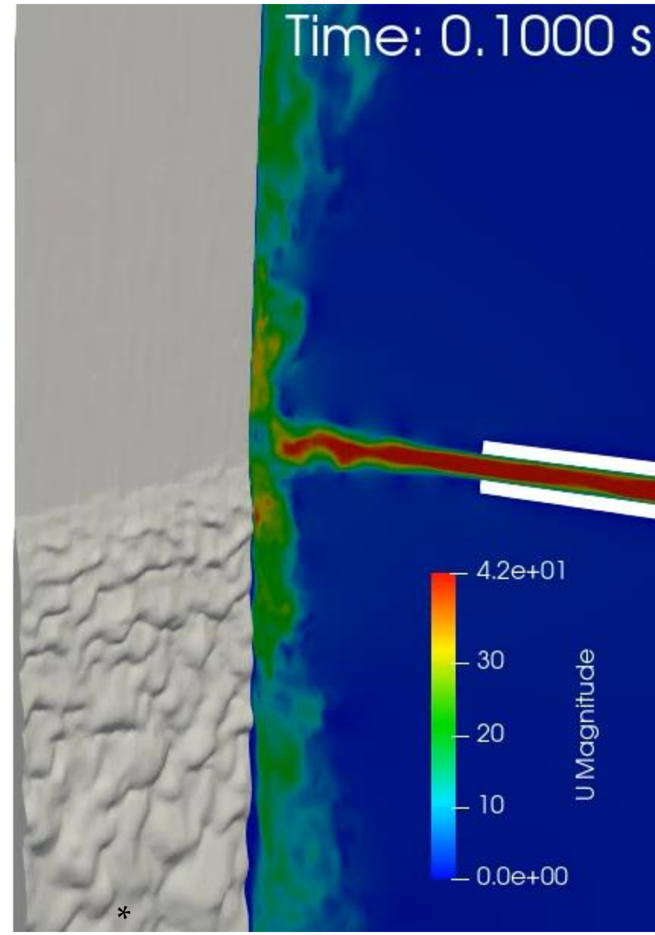
The CFD overestimates the damping of the undulation due to mesh grading towards the outlet of the domain.



What about jet wiping with zinc?

$\Delta P_N = 1{,}5\ kPa \mid U_p = 1\ m/s \mid Z = 10\ mm \mid d = 1\ mm$





Water

$\rho_l = 1000\ kg/m^3$
$\mu_l = 0{,}001\ Pa \cdot s$
$\sigma_l = 0{,}072\ N/m$

Zinc

$\rho_l = 6400\ kg/m^3$
$\mu_l = 0{,}0046\ Pa \cdot s$
$\sigma_l = 0{,}7\ N/m$

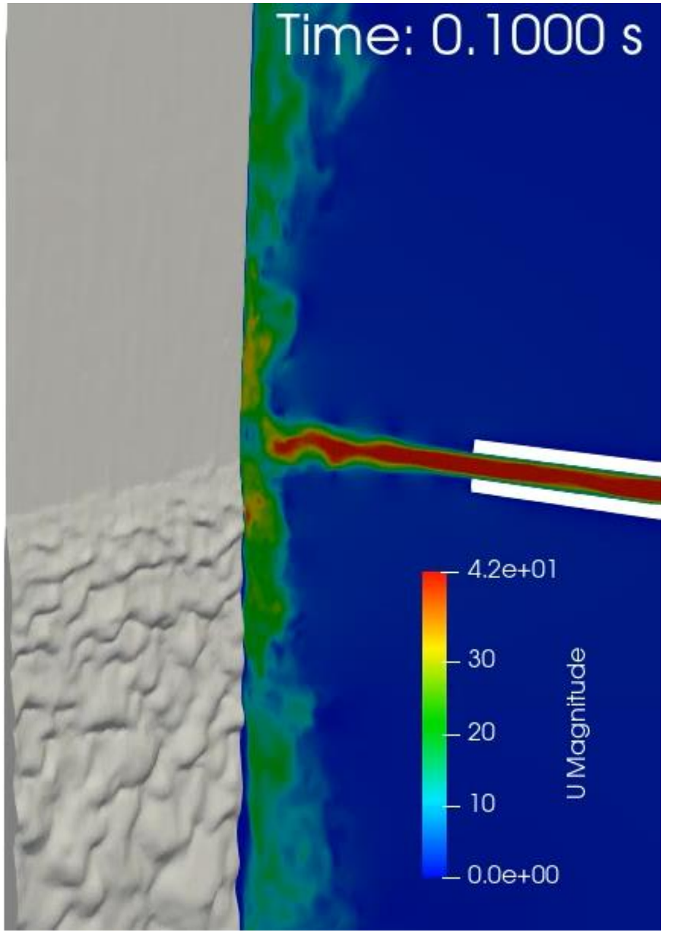
What about jet wiping with zinc?

$\boldsymbol{\Delta P_N = 5\ kPa} \mid \boldsymbol{U_p = 2m/s} \mid Z = 10\ mm \mid d = 1\ mm$

	Dipropylene	Water	Attempt zinc	Galvanization
h_f	450 μm	20 μm	33.6 μm	8 μm
h_0	1594 μm	320 μm	302 μm	213 μm
h_f/h_0	0.28	0.063	0.11	0.038
h_0/Z	0.089	0.032	0.030	0.018

Crash!*

Computational resources:
- 18M cells
- 1024 processors
- Time step ≈ 10^{-7} s
- 5 h CPU / ms of real flow
- F_s = 10 kHz
- 5 GB / ms of real flow



Zinc

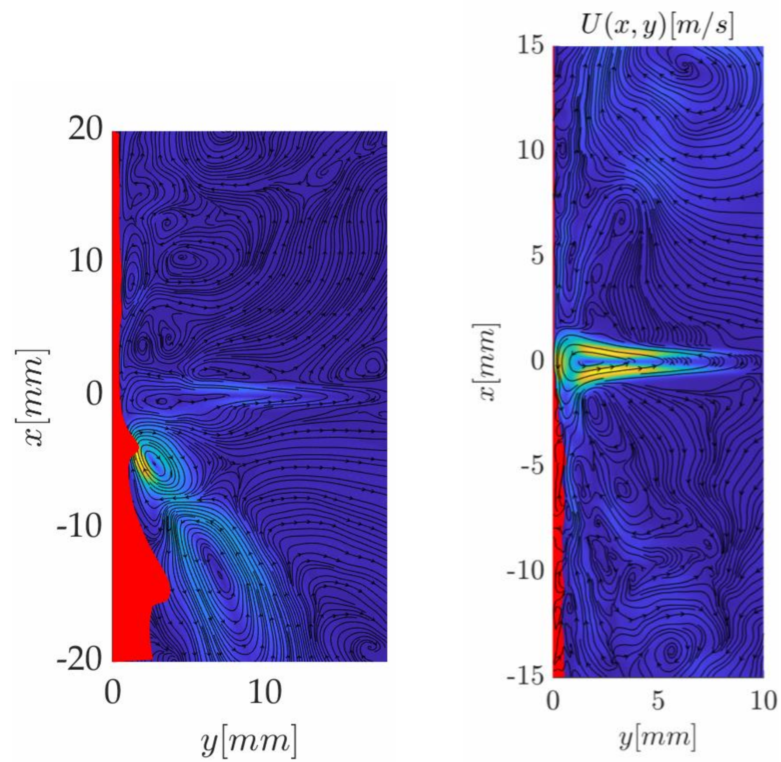
Conclusions and future work

Research Questions

1) **What is the mechanism responsible for undulation in improved similarity conditions?**

2) Is it possible to simulate jet wiping using integral models?

The undulation dynamics is different compared to the lab-scale conditions with dipropylene glycol



- Lower *intrusivity* of the liquid film
- Fully developed run-back flow.
- Oscillation of the gas jet.

Conclusions and future work

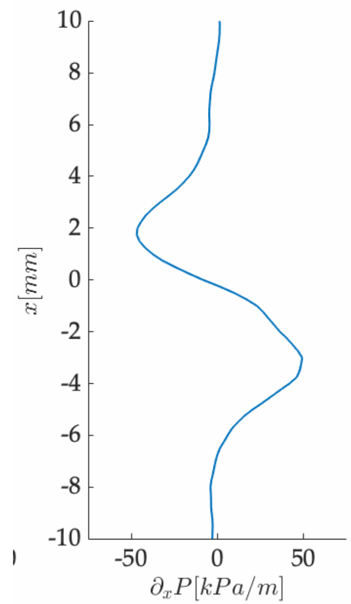
Research Questions

1) **What is the mechanism responsible for undulation in improved similarity conditions?**

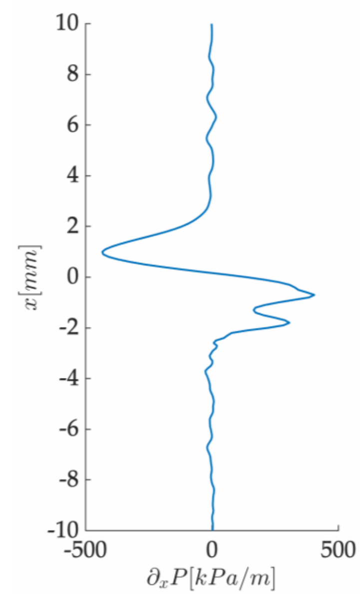
2) Is it possible to simulate jet wiping using integral models?

Nevertheless, the main undulation pattern and the pressure gradient evolution are similar (although at higher frequency), suggesting that **the mechanism is the same!**

Dipropylene glycol



Water



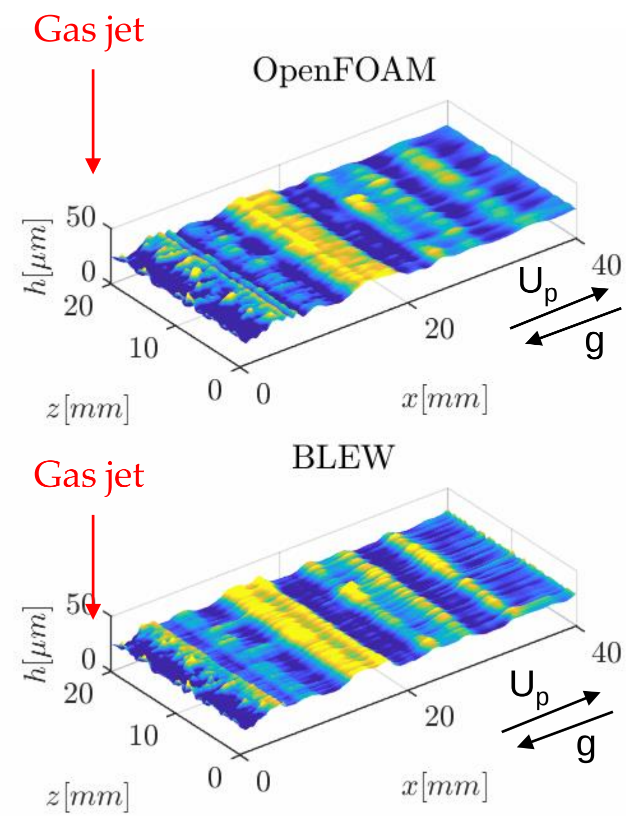
Conclusions and future work

Research Questions

1) What is the mechanism responsible for undulation in improved similarity conditions?

2) Is it possible to simulate jet wiping using integral models?

- We have **validated** the **integral formulation for jet wiping**. **It is the only viable method to tackle jet wiping in industrial conditions.**
- Can be applied for flow control in industrial lines.



Conclusions and future work

High-fidelity CFD (OpenFOAM)

✓
- Captures the two-way coupling.
- Realistic data from the gas jet.

✗
- High computational cost.
- Numerical problems at the interface: diffusion, turbulence and parasitic currents.

Low-fidelity modelling (Integral model)

✓
- Modest computational cost.
- Represents well the dynamics of the film.

✗
- The quality of the results depends on the quality of the inputs: jet actuators.
- Unable to tackle extreme process conditions (i.e.. splashing).

Conclusions and future work

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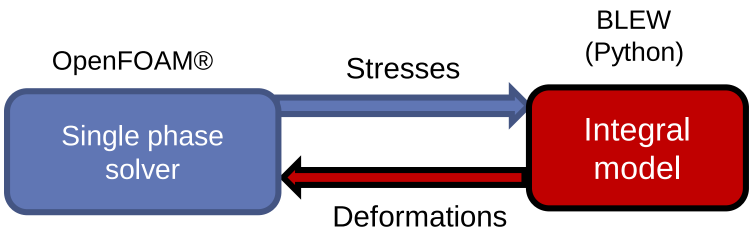
What if we combine the best of the two worlds?

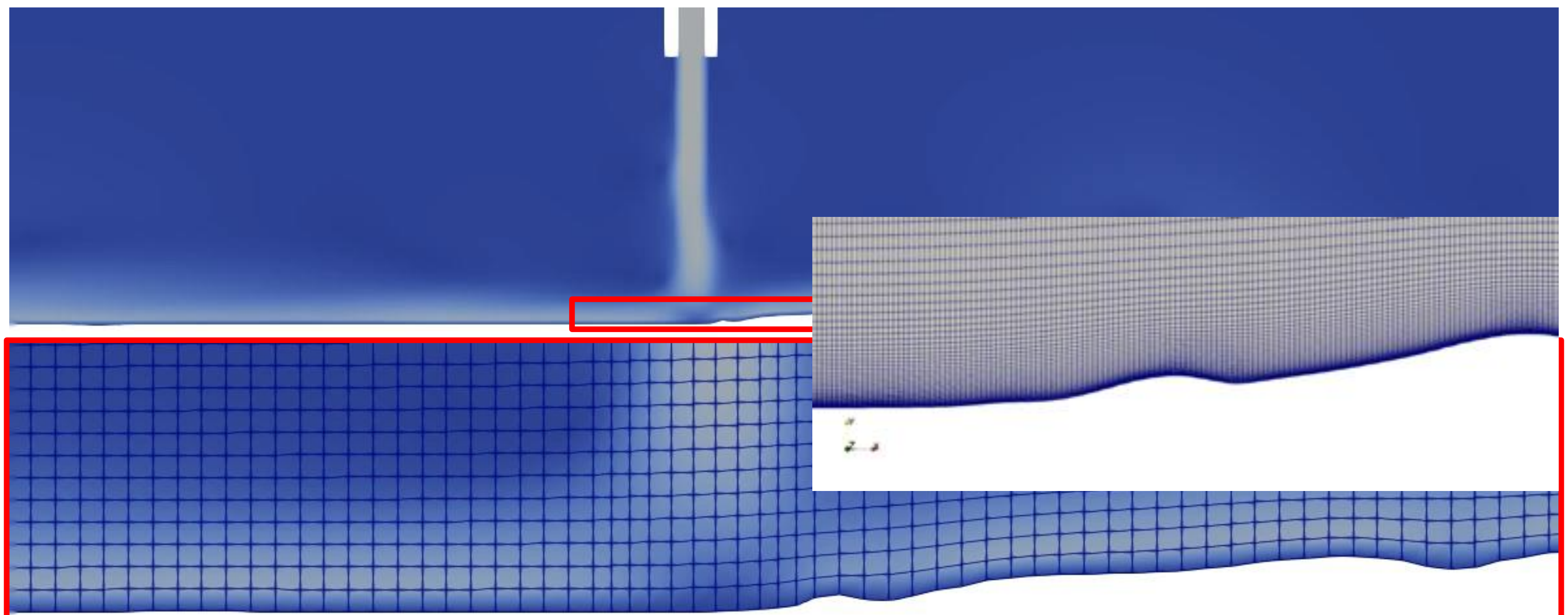
Conclusions and future work

Hybrid model

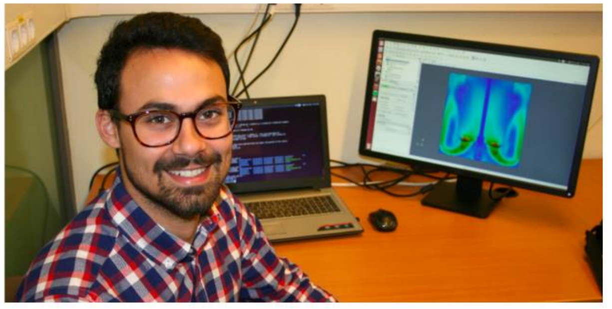


Coupling software

preCICE



The team



David Barreiro Villaverde
PhD student

Dr. Marcos Lema
Assistant Prof.
Universidade da Coruña

Dr. Miguel A. Mendez
Assistant Prof.
Von Karman Institute for Fluid Dynamics

Thank you for your attention

Anne Gosset

D. Barreiro-Villaverde[2,3], M.A Mendez[3] and M. Lema[2]

[1] CITENI, Campus Industrial de Ferrol
Universidade da Coruña, Spain

[2] CITIC Research, **Universidade da Coruña**, Spain
[3] **von Karman Institute**, Sint-Genesius-Rode, Belgium

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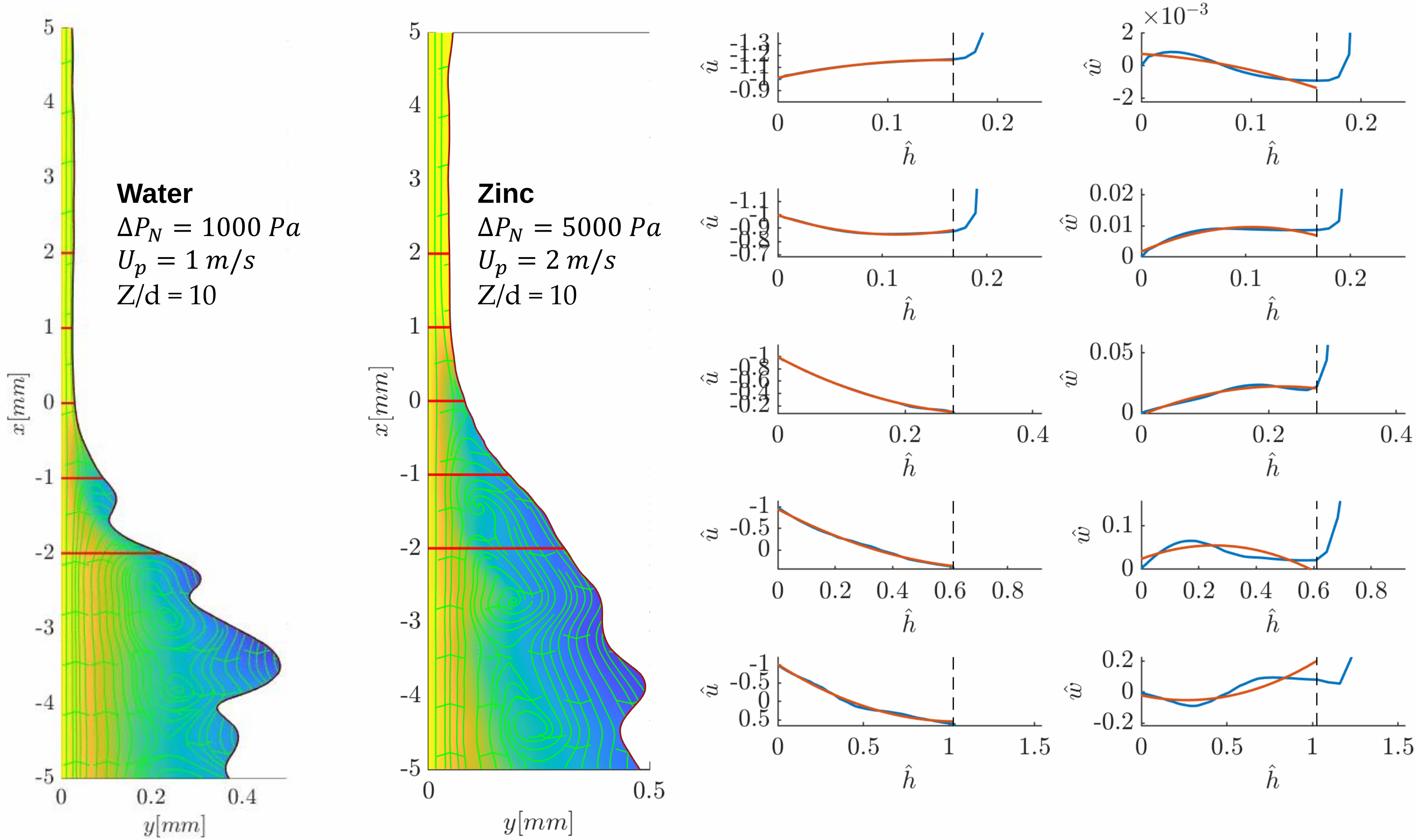
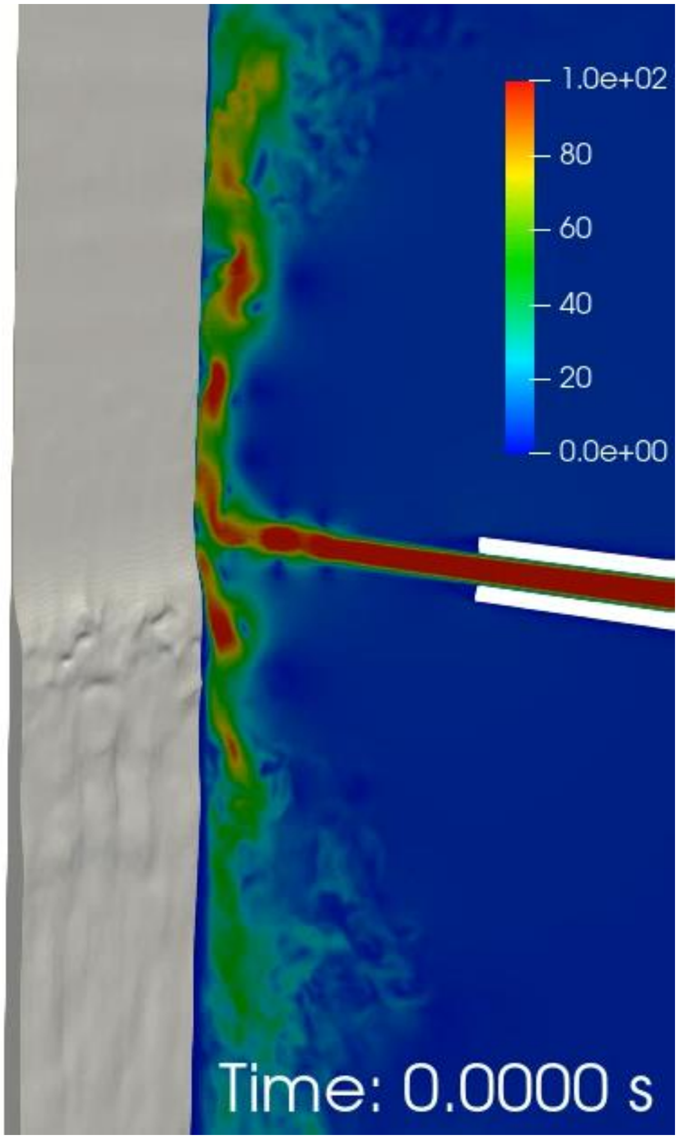
RES User's Conference, Cáceres. 13-14th September 2022

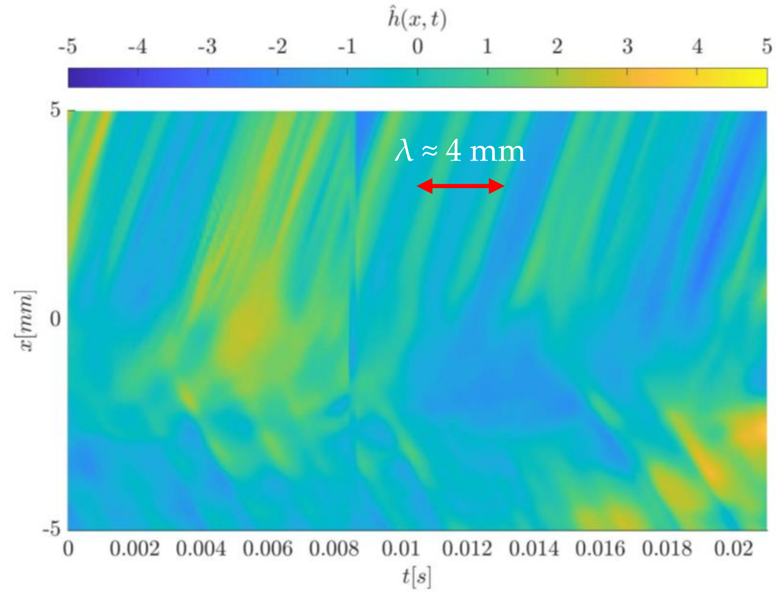
What about jet wiping with zinc?



Parabolic assumption ✓
Dynamic similarity between water and zinc ✓

What about jet wiping with zinc?





In the short time window computed we can observe the formation of two-dimensional undulations of wavelength $\lambda \approx 4$mm.

The amount of data does not allow for statistical or mPOD analysis.

Test al methodology

One fluid, two-phase NSE

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0$$

$$F_{st} = \sigma\, \kappa\, \nabla \alpha$$

$$\frac{\partial (\rho \boldsymbol{v})}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}\boldsymbol{v}) = -\nabla p + \nabla \cdot \tau + \rho g + F_{st}$$

$$\frac{\partial \alpha}{\partial t} + \nabla \cdot (\alpha \boldsymbol{v}) = 0 \longrightarrow \textbf{Volume of Fluid (VOF)}$$

α : liquid volume fraction

$$\begin{cases} \alpha = 0 \rightarrow \text{gas} \\ 0 < \alpha < 1 \rightarrow \text{free surface} \\ \alpha = 1 \rightarrow \text{liquid} \end{cases}$$

$$\phi = \phi_l \alpha + \phi_g (1 - \alpha)$$

$$\rho \frac{\partial \boldsymbol{U}}{\partial t} +$$

$\nabla \cdot \mathbf{U} = 0,$

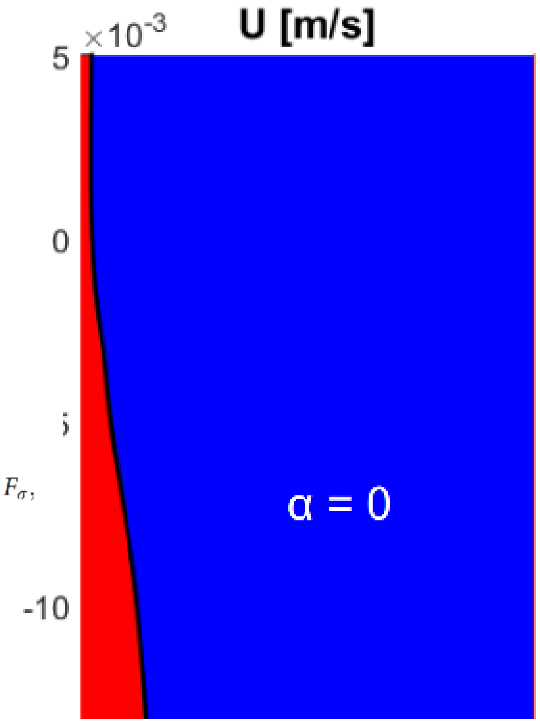
$-\nabla p + \rho \mathbf{g} + \nabla \cdot [2\mu \mathbf{S}] + F_\sigma,$

Volume of Fluid (VOF)

$$\frac{\partial \alpha}{\partial t} + \nabla \cdot (\alpha \boldsymbol{U}) + \underbrace{\nabla \cdot (\boldsymbol{U_r}\, \alpha (1 - \alpha))}_{\text{Interface compression}} = 0$$

$$F_\sigma = \sigma \kappa \boldsymbol{n} \delta$$

$$\frac{\partial \alpha}{\partial t} + \nabla \cdot (\alpha \mathbf{U}) = 0.$$



$$\nabla \cdot \bar{\mathbf{U}} = 0,$$

$$\frac{\partial (\bar{\rho} \bar{\mathbf{U}})}{\partial t} + (\bar{\rho} \bar{\mathbf{U}} \cdot \nabla) \bar{\mathbf{U}} = -\nabla \bar{p} + \rho \mathbf{g} + \nabla \cdot [2 \bar{\mu} \bar{\mathbf{S}}] + \sigma \bar{\kappa} \nabla \bar{\alpha} + \nabla \cdot (\tau_\mu - \tau_u) + \tau_\sigma,$$

$$\frac{\partial \bar{\alpha}}{\partial t} + \nabla \cdot (\bar{\mathbf{U}} \bar{\alpha}) = \nabla \cdot \tau_\alpha.$$

Large Eddy Simulation (LES)

$\tau_u = -2\mu_t \bar{S}$: sub-grid advection term

$\mu_t = \rho (C_s \Delta x_{cell})^2 |\bar{S}|$: turbulent viscosity

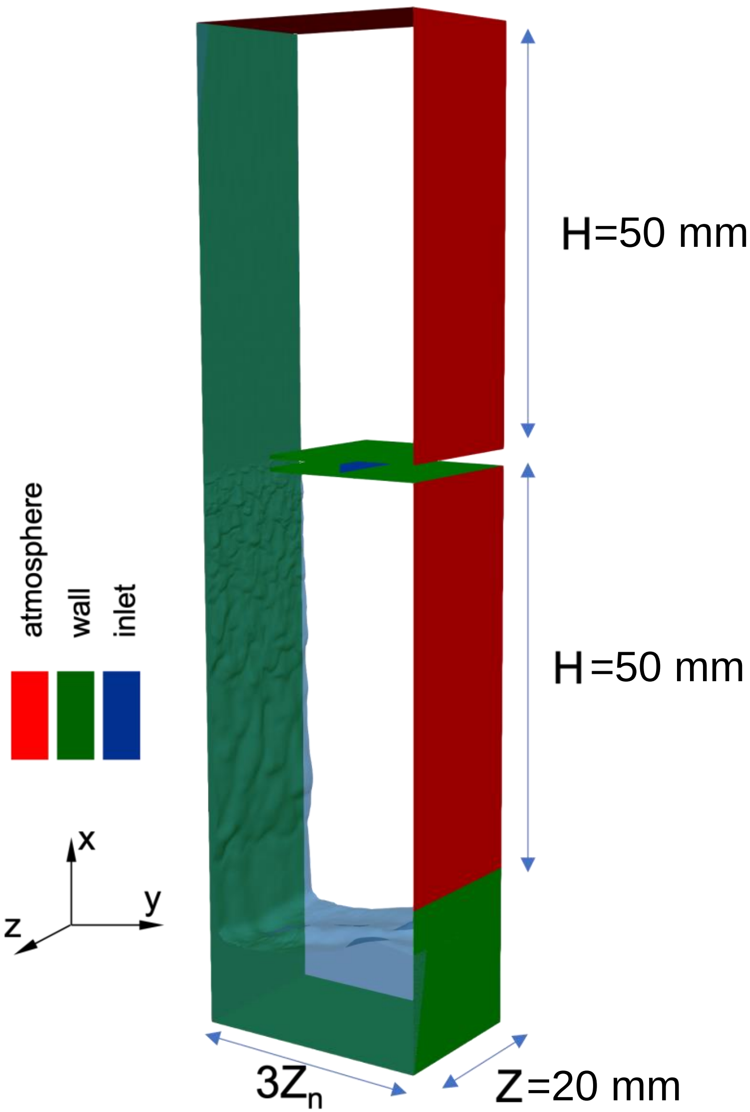
$|\bar{S}| = \sqrt{2 \bar{S}\, \bar{S}}$: strain rate tensor (resolved)

Several subgrid terms arise in the filtered equations: the adve
SGS term, τ_u (also present in single phase LES, and given by Eq.
in Lakehal *et al.*[48]), is accounted for using the Smagorinsky m
with an eddy viscosity approximation,

$$\tau_u = -2\mu_t \bar{\mathbf{S}},$$

in which the turbulent eddy viscosity μ_t reads

Test case and numerical methodology



$$\Delta P_N = 1\,kPa$$
$$U_p = 1\,m/s$$
$$Z = 10\,mm$$
$$d = 1\,mm$$

Computational cost

- **1500 - 2200 h CPU/s** of real flow using 512 processors (Intel Xeon E5-2680v3).
- **>3M CPU hours**.
- **10 TB** of high-quality data.